MAST602: Introduction to Physical Oceanography (Andreas Münchow)
(Closed book in-class Rossby Wave Exercise, Oct.-21, 2008)

In a series of papers published in the 1930ies Carl-Gustaf Rossby introduced a strange new wave form whose existence has not been confirmed observationally until the late 1990ies. A debate is presently raging if and how these waves may impact ecosystems in the ocean, e.g., "*Killworth et al., 2003: Physical and biological mechanisms for planetary waves observed in satellite-derived chlorophyll, J. Geophys. Res.*" and "*Dandonneau et al., 2003: Oceanic Rossby waves acting as a "Hay Rake" for ecosystem floating by-products, Science*" as well as a flurry of comments generated by these papers.

These peculiar waves originate from a linear balance between local acceleration, Coriolis acceleration, and pressure gradients as well as continuity of mass, that is,

East-west momentum balance:	$\partial u/\partial t - fv = -g\ \partial\eta/\partial x$
North-south momentum balance:	$\partial v/\partial t + fu = -g\ \partial\eta/\partial y$
Continuity:	$\partial\eta/\partial t + H\ (\partial u/\partial x+\partial v/\partial y) = 0$

where the Coriolis parameter $f = 2\ \Omega\ \sin(\text{latitude})$ is no longer a constant, but is approximated locally as $f \approx f_0 + \beta y$. The rotational rate of the earth is $\Omega=2\pi/\text{day}$ and at the latitude of Lewes, DE (39N), $f_0 \sim 0.9\times10^{-4}\ s^{-1}$ and $\beta \sim 2\times10^{-11}\ m^{-1}\ s^{-1}$.

A number of peculiar properties can be inferred from its dispersion relation:

$$\sigma = -\ \beta\ \kappa\ /\ (\kappa^2+l^2+R^{-2})$$

where σ is the wave frequency, κ is the wave number in the east-west direction, l is the wave number in the north-south direction, β is a constant (the so-called beta-parameter that incorporates Coriolis effects that changes with latitude), and $R=(g'H)^{1/2}/f_0$ is a constant (the so-called Rossby radius of deformation, the same as the lateral decay scale of the Kelvin wave, where $g' \sim 9.81\times10^{-3} m/s^2$ is the constant of "reduced" gravity, and $H \sim 1000$ m is the depth of the pycnocline (density interface).

Please note that the wavelength has components in both horizontal directions, but that without loss of generality we may consider an east-west propagating wave by setting $l=0$. Still the resulting dispersion relation is most peculiar:

$$\boldsymbol{\sigma = -\ \beta\ \kappa\ /\ (\kappa^2+R^{-2})}$$

Your task here is to **describe as many characteristics of this wave as possible** based on this dispersion relation. Note that its wavelength $\lambda=2\pi/\kappa$ compares against a horizontal length scale R. Therefore, you can distinguish between Rossby waves that are long and short relative to R. This is somewhat analogous to the way we distinguished between deep of shallow surface gravity water waves.
[Key words: wavelength, phase velocity, group velocity, direction of propagation, dispersion, short waves, long waves, Rossby radius, time and space scales]