10

Internal Waves

Summary: Internal gravity waves supported by vertical stratification are explored. After the derivation of the dispersion relation and an examination of wave properties, the chapter concludes with a discussion of mountain waves.

10-1 FROM SURFACE TO INTERNAL WAVES

Starting at an early age, everyone has seen, experienced, and wondered about surface waves. Sloshing of water in bathtubs and kitchen sinks, ripples on a pond, surf at the beach, and swell further offshore are all manifestations of surface water waves. Sometimes we look at them with disinterest, and sometimes they fascinate us. But, whatever our reaction or interest, their mechanism relies on a simple balance between gravity and inertia. When the surface of the water is displaced upward, gravity pulls it back downward, the fluid develops a vertical velocity (potential energy turns into kinetic energy) and, because of inertia, the surface penetrates below its level of equilibrium. An oscillation results. A change in the phase of the oscillations from place to place causes the wave to travel. Because surface waves carry energy and no volume, they naturally occur wherever there is agitation that causes no overall water displacements, such as the shaking of a half-full bottle, the throwing of a stone in a pond, or a storm at sea.

The gravitational force continuously strives to restore the water surface to a horizontal level, because the water density is greater than that of the air above. It goes almost without saying that the same mechanism is at work whenever two fluid densities differ. This frequently occurs in the atmosphere when warm air overlies cold air; waves may then be manifested by cloud undulations, which may at times be remarkably periodic (Figure 10-1). An oceanic example, known as the phenomenon of dead water

Figure 10-1 Evidence of internal wave in the atmosphere. The presence of moisture causes condensation in the rising air (wave crests), thus revealing the internal wave as a periodic succession of cloud bands. (Photo by the author.)

(Figure 1-3), is the occurrence of waves at the interface between an upper layer of relatively light water and a denser lower layer. Those waves, although unseen from the surface, can cause a sizable drag on a sailing vessel (Section 1-3).

But the existence of such interfacial waves is not restricted to fluids with two distinct densities and a single interface. With three densities and two interfaces, two wave modes are possible; if the middle layer is relatively thin, the vertical excursions of the interfaces interact, letting energy pass from one level to the other. At the limit of a continuously stratified fluid, an infinite number of modes is possible, and wave propagation has both horizontal and vertical components (Figure 10-2). Regardless of the level of apparent complexity in the wave pattern, the mechanism remains the same:

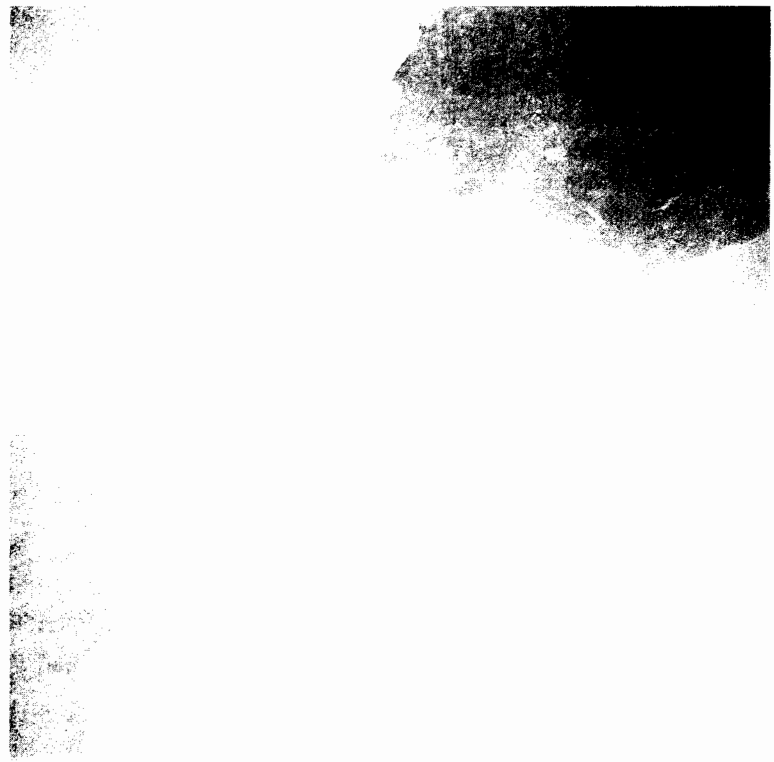

Figure 10-2 Surface manifestation of oceanic internal waves. The upward energy propagation of internal waves modifies the properties of surface waves. In this sunglint photograph taken from the space shuttle *Discovery*, two groups of internal waves are visible, one in the center of the Strait of Gibraltar and the other, having been produced during the preceding diurnal tide, propagating east into the Mediterranean Sea. (NASA Photo # STS-42-075-015, courtesy of Steven G. Ackleson, Lockheed Engineering and Science Company, 2400 NASA Rd. 1, Houston, TX 77258.)

There is a continuous interplay between gravity and inertia and a continuous exchange between potential and kinetic energy.

10-2 INTERNAL-WAVE THEORY

To study internal waves in their purest form, a few assumptions are necessary: There is no ambient rotation, the domain is infinite in all directions, there is no dissipative mechanism of any kind, and, finally, the fluid motions and wave amplitudes are small. The last assumption is made to permit the linearization of the governing equations. However, we reinstate a term previously neglected, namely, the vertical acceleration term $\partial w/\partial t$ in the vertical momentum equation. We do so anticipating that vertical accelerations may play an important role in gravity waves. (Recall the discussion in Section 9-2 on the vertical oscillations of fluid parcels in a stratified fluid, which included the vertical acceleration.) The inclusion of this term breaks the hydrostatic balance, but so be it! Finally, we clarify the expression for the fluid density by writing

$$\text{Actual fluid density} = \rho_0 + \bar{\rho}(z) + \rho'(x, y, z, t), \tag{10-1}$$

where ρ_0 is the reference density (a pure constant), $\bar{\rho}(z)$ is the ambient equilibrium stratification, and $\rho'(x, y, z, t)$ is the density fluctuation induced by the wave (lifting and lowering of the ambient stratification). The inequality $|\bar{\rho}| \ll \rho_0$ is enforced to justify the Boussinesq approximation (Section 3-3), whereas the further inequality $|\rho'| \ll |\bar{\rho}|$ is required to linearize the wave problem. The total pressure field can be decomposed in a similar manner.

With the preceding assumptions, the governing equations become (Section 3-5)

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial x}, \tag{10-2}$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial y}, \tag{10-3}$$

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial z} - \frac{1}{\rho_0} g\rho', \tag{10-4}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{10-5}$$

$$\frac{\partial \rho'}{\partial t} + w\frac{d\bar{\rho}}{dz} = 0. \tag{10-6}$$

The factor $d\bar{\rho}/dz$ in the last term can be transformed by introducing the stratification frequency (alias the Brunt–Väisälä frequency) defined in the preceding chapter by (9-3):

$$N^2 = -\frac{g}{\rho_0}\frac{d\bar{\rho}}{dz}. \tag{10-7}$$

For simplicity, we will assume it to be uniform over the extent of the fluid. This corresponds to a linear density variation in the vertical. Because all coefficients in the preceding linear equations are constant, a wave solution of the form

$$e^{i(lx + my + nz - \omega t)}$$

is sought. Transformation of the derivatives into products (e.g., $\partial/\partial x = il$) leads to a 5-by-5 homogeneous algebraic problem. The solution is nonzero if the determinant vanishes, and this requires that the wave frequency ω be given by

$$\omega^2 = N^2 \frac{l^2 + m^2}{l^2 + m^2 + n^2} \tag{10-8}$$

in terms of the wave numbers, l, m, and n, and the stratification frequency, N. This is the dispersion relation of internal gravity waves.

A number of wave properties can be stated by examination of this relation. First and foremost, it is obvious that the numerator is always smaller than the denominator, meaning the wave frequency will never exceed the stratification frequency; that is,

$$\omega \leq N \tag{10-9}$$

for positive frequencies. The reason for this upper bound can be traced back to the presence of the vertical acceleration term in (10-4). Indeed, without that term the denominator in (10-8) reduces from $l^2 + m^2 + n^2$ to only n^2, implying that the non-hydrostatic term can be neglected as long as $l^2 + m^2 \ll n^2$. This occurs for waves with horizontal wavelengths much longer than their vertical wavelengths; the frequency of those waves is much less than N. For progressively shorter waves, the correction becomes increasingly important, the frequency rises but saturates at the value N. We may then ask what would happen if we agitate a stratified fluid at a frequency greater than its own stratification frequency. The answer is that, with such short periods, particles do not have the time to oscillate at their natural frequency and instead follow whatever displacements are forced upon them; the disturbance is local, and no energy is carried away by waves.

Another important property derived from the dispersion relation (10-8) is that the frequency does not depend on the wave-number magnitude (and thus on the wavelength) but only on its angle with respect to the horizontal plane. Indeed, noting that $l = k \cos\theta \cos\phi$, $m = k \cos\theta \sin\phi$, and $n = k \sin\theta$, where $k = (l^2 + m^2 + n^2)^{1/2}$ is the wave-number magnitude, θ is its angle from the horizontal (positive or negative), and ϕ is the angle of its horizontal projection with the x-axis, we obtain

$$\omega = \pm N \cos\theta, \tag{10-10}$$

proving that the frequency depends only on the pitch of the wave number, and, of course, the stratification frequency. The fact that two signs are allowed indicates that the wave can travel in one of two directions, upward or downward along the wave-number direction. On the other hand, if the frequency is imposed, all waves, regardless of wavelength, propagate at fixed angles from the horizontal. The lower the frequency, the steeper the direction. At the limit of very low frequencies, the phase propagation is purely vertical ($\theta = 90°$).

10-3 STRUCTURE OF AN INTERNAL WAVE

Let us rotate the x- and y-axes so that the wave-number vector is contained in the (x, z) vertical plane (i.e., $m = 0$ and there is no variation in the y-direction and no v velocity component). The solutions for the remaining two velocity components and the density fluctuation are

$$u = U \sin(lx + nz - \omega t), \tag{10-11a}$$

$$w = -\frac{l}{n} U \sin(lx + nz - \omega t), \tag{10-11b}$$

$$\rho' = -A \cos(lx + nz - \omega t), \tag{10-11c}$$

where the amplitudes are related by

$$A = \frac{\rho_0 N}{g} \frac{\sqrt{l^2 + n^2}}{n} U. \tag{10-11d}$$

Let us take l, n, and ω all positive, which is the situation depicted on Figure 10-3. The areas of upwelling (crests) and downwelling (troughs) alternate both horizontally and

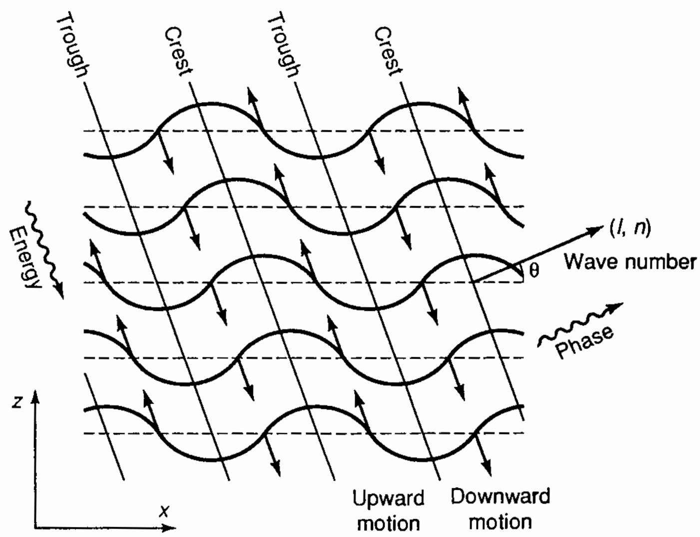

Figure 10-3 Vertical structure of an internal wave.

vertically, and lines of constant phase (e.g., following crests) tilt perpendicularly to the wave-number vector. The trigonometric functions in solution (10-11) tell us that the phase $lx + nz - \omega t$ remains constant with time if one translates in the direction (l, n) of the wave number at the speed (see Appendix A):

$$c = \frac{\omega}{\sqrt{l^2 + n^2}}. \tag{10-12}$$

This is the phase speed, at which lines of crests and troughs translate. Because the velocity components, u and w, are in quadrature with the density fluctuations, the velocity is nil at the crests and troughs but is maximum a quarter of a wavelength away. The signs indicate that when one component is positive, the other is negative, implying downwelling to the right and upwelling to the left, as indicated in Figure 10-3. The ratio of velocities ($-l/n$) further indicates that the flow is everywhere perpendicular to the wave-number vector and thus parallel to the lines connecting crests and troughs. Internal waves are transverse waves. A comparison of the signs in the expressions of w and ρ' reveals that rising motions occur ahead of crests and sinking motions occur ahead of troughs, eventually forming the next crests and troughs, respectively. Thus, the wave moves forward and, because of the inclination of its wave number, also upward.

The propagation of the energy is given by the group velocity, which is the gradient of the frequency with respect to the wave number (Appendix A):

$$c_{gx} = \frac{\partial \omega}{\partial l} = \frac{\omega n^2}{l(l^2 + n^2)} \tag{10-13a}$$

$$c_{gz} = \frac{\partial \omega}{\partial n} = -\frac{\omega n}{(l^2 + n^2)}. \tag{10-13b}$$

The direction is perpendicular to the wave number (l, n) and is downward. Thus, although the crests and troughs appear to move upward, the energy actually sinks. The reader can verify that, irrespective of the signs of the frequency and wave-number components, the phase and energy always propagate in the same horizontal direction (though not at the same rates) and in opposite vertical directions.

Let us now turn our attention to the extreme cases. The first one is that of a purely horizontal wave number ($n = 0$, $\theta = 0$). The frequency is then N, and the phase speed is N/l. The absence of wavelike behavior in the vertical direction implies that all crests and troughs are vertically aligned. The motion is strictly vertical, and the group velocity vanishes, implying that the energy does not travel. The opposite extreme is that of a purely vertical wave number ($l = 0$, $\theta = 90°$). The frequency vanishes, implying a steady state. There is then no wave propagation. The velocity is purely horizontal and, of course, laterally uniform. The picture is that of a stack of horizontal sheets each moving, without distortion, with its own speed and in its own direction. If a boundary obstructs the flow at some depth, none of the fluid at that depth, however remote from the obstacle, is allowed to move. This phenomenon, occurring at very low frequencies in highly stratified fluids, is none other than the blocking phenomenon discussed at the end of Section 9-4 and presented as the stratified analogue of the Taylor column in rotating fluids.

In stratified and rotating fluids, the lowest possible internal-wave frequency is not zero but the inertial frequency f (see Problem 10-3). At that limit, the wave motion assumes the form of inertial oscillations, wherein fluid parcels execute horizontal circular trajectories (Section 2-4). Such limiting behavior is an attribute of inertia-gravity waves in homogeneous rotating fluids (Section 6-3) and is not surprising, since internal waves in stratified rotating fluids are the three-dimensional extensions of the inertia-gravity waves of homogeneous rotating fluids.

10-4 LEE WAVES

Internal waves in the atmosphere and ocean can be generated by myriad processes, almost wherever a source of energy has some temporal or spatial variability. Oceanic examples include the ocean tide over a sloping bottom, mixing processes in the upper ocean (such as during a hurricane), instabilities of shear flows, and the passage of a submarine. In the atmosphere, one particularly effective mechanism is the generation of internal waves by a wind blowing over an irregular terrain such as a mountain range or a hilly countryside. We select the latter example to serve as an illustration of internal-wave theory because it has some meteorological importance and lends itself to a simple mathematical treatment.

To apply the previous linear-wave theory, we naturally restrict our attention to small-amplitude waves and, consequently, to small topographic irregularities. This restriction also permits us to study a single topographic wavelength, from which the principle of linear superposition will allow us to construct more general solutions. The model (Figure 10-4) consists of a stratified air mass of uniform stratification frequency

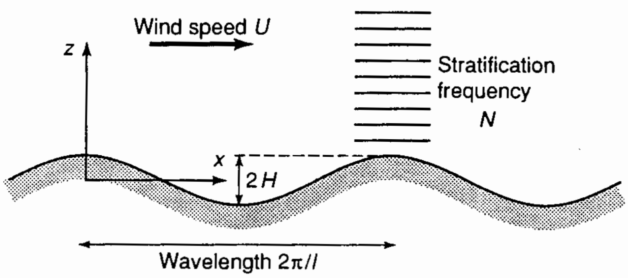

Figure 10-4 Stratified flow over a wavy terrain. The difference in elevation between crests and troughs is assumed small to justify a linear analysis. Then, the flow over any terrain configuration can be obtained from the superimposition of elementary wave solutions.

N flowing at speed U over a slightly wavy terrain. The terrain elevation is taken as a sinusoidal function $h = H \cos lx$ of amplitude H (the trough-to-crest height difference is then $2H$) and wave number l (the wavelength is then $2\pi/l$). The wind direction (along the x-axis of the model) is chosen to be normal to the troughs and crests, so that the problem is two-dimensional.

Because our theory has been developed for waves in the absence of a main flow, we translate the x-axis at the wind speed. The topography then appears to move at speed U in the negative x-direction:

$$\begin{aligned} z = h(x + Ut) &= H \cos[l(x + Ut)] \\ &= H \cos(lx - \omega t), \end{aligned} \tag{10-14}$$

where the frequency is defined as

$$\omega = -lU \tag{10-15}$$

and is a negative quantity. Because a particle initially on the bottom must remain there at all times (no airflow through terrain), a boundary condition is

$$w = \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} \quad \text{at } z = h,$$

which can be immediately linearized to become

$$w = \frac{\partial h}{\partial t}$$

$$= H\omega \sin(lx - \omega t) \quad \text{at } z = 0, \qquad (10\text{-}16)$$

by virtue of our small-amplitude assumption.

The solution to the problem, which must simultaneously be of type (10-11) and meet condition (10-16), can be readily formulated:

$$u = nUH \sin(lx + nz - \omega t), \qquad (10\text{-}17a)$$

$$w = -lUH \sin(lx + nz - \omega t), \qquad (10\text{-}17b)$$

$$p' = -\rho_0 nU^2 H \sin(lx + nz - \omega t), \qquad (10\text{-}17c)$$

$$\rho' = \frac{\rho_0 N^2 H}{g} \cos(lx + nz - \omega t), \qquad (10\text{-}17d)$$

where the vertical wave number n is determined by the dispersion relation (10-8):

$$n^2 = \frac{N^2}{U^2} - l^2. \qquad (10\text{-}18)$$

The mathematical structure of this last expression shows that two cases must be distinguished: Either N/U is larger than l and n is real, or N/U is smaller than l and n is imaginary.

10-4-1 Radiating Waves

Let us first explore the former situation, which arises when the stratification is sufficiently strong ($N > lU$) or when the topographic wavelength is sufficiently long ($l < N/U$). Physically, the time $2\pi/lU$ taken by a particle traveling at the mean wind speed U to go from a trough to the next trough (i.e., up and down once) is longer than the natural oscillatory period $2\pi/N$, and internal waves can be excited. Solving (10-18) for n, we have two solutions at our disposal,

$$n = \pm\sqrt{\frac{N^2}{U^2} - l^2},$$

but because the source of wave energy is at the bottom, only the wave with upward group velocity is physically relevant. According to (10-13b) and (10-15), we select the positive root.

The wave structure in the framework fixed with the earth (Figure 10-5) is steady and such that all density surfaces undulate like the terrain, with no vertical attenuation

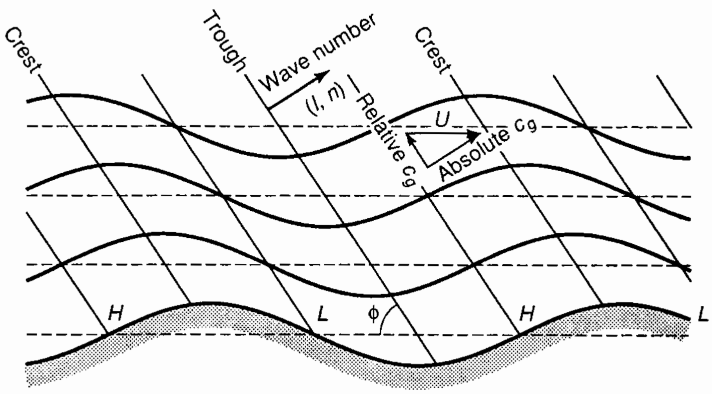

Figure 10-5 Structure of the mountain wave in the case of strong stratification or long wavelength ($N > lU$). Note the absence of vertical attenuation and the presence of a phase shift with height. The group velocity with respect to the ground is oriented upward and downwind. The pressure distribution, with highs on wind-facing slopes and lows on flanks in the wind's shadow, exerts a drag on the moving air mass.

but with an upwind phase tilt with height. The tilt angle between the wave fronts (lines joining crests) and the horizontal, ϕ, is given by

$$\sin\phi = \frac{lU}{N}, \tag{10-19}$$

so that $l = k \sin\phi$, $n = k\cos\phi$, with $k = (l^2 + n^2)^{1/2}$. The group velocity in the fixed frame is equal to the group velocity relative to the moving wind, given by (10-13) with $\omega = -lU$, plus the velocity U in the x-direction:

$$c_{gx} = -U\frac{n^2}{k^2} + U = U\sin^2\phi \tag{10-20a}$$

$$c_{gz} = U\frac{ln}{k^2} = U\sin\phi\cos\phi. \tag{10-20b}$$

It tilts upward as required, and its direction coincides with that of the wave number (Figure 10-5). Energy is thus radiated upward and downwind. We shall not calculate the energy flux and will show only that the terrain exerts a drag force on the flowing air mass. The Reynolds-stress expression for the wave stress is

$$\text{Drag force} = -\rho_0\,\overline{uw}\,|_{z=0} = -\tfrac{1}{2}\,\rho_0 lnU^2H^2,$$

where the overbar indicates an average over one wavelength. The minus sign indicates a retarding force. The existence of this force is also related to the fact that the high pressures are situated on the hill flanks facing the wind, and the lows are on the hill flanks in the wind's shadow.

10-4-2 Trapped Waves

The second case, leading to an imaginary value for n, occurs for weak stratifications ($N < lU$) or short waves ($l > N/U$). To avoid dealing with imaginary numbers we define the quantity a as the positive imaginary part of n, that is, $n = \pm ia$ with

$$a = \sqrt{l^2 - \frac{N^2}{U^2}}. \tag{10-21}$$

The solution now contains exponential functions in z, and the physical nature of the problem dictates that we retain only the function that decays away from the ground. In the reference framework translating with the wind speed U, the solution is

$$u = aUH\, e^{-az} \cos(lx - \omega t), \tag{10-22a}$$

$$w = -lUHe^{-az} \sin(lx - \omega t), \tag{10-22b}$$

$$p' = -\rho_0 aU^2He^{-az} \cos(lx - \omega t), \tag{10-22c}$$

$$\rho' = \frac{\rho_0 N^2 H}{g} e^{-az} \cos(lx - \omega t). \tag{10-22d}$$

The wave structure is depicted in Figure (10-6). Density surfaces undulate at the same wavelength as the terrain, but the amplitude decays with height. There is also no vertical phase shift. Because the waves are contained near the ground, in a boundary layer of thickness on the order of $1/a$, there is no upward energy radiation. The absence of such energy loss is corroborated by the absence of a drag force:

$$\text{Drag force} = -\rho_0 \overline{uw}\,|_{z=0} = 0.$$

The Reynolds stress vanishes because u and w are now in quadrature. Physically, the high pressures are in the valleys, the lows are on the hilltops, and the pressure distribution causes no work against the wind.

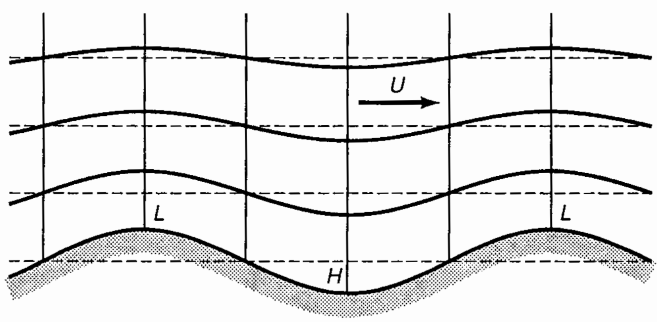

Figure 10-6 Structure of the mountain wave in the case of weak stratification or short wavelength ($N < lU$). Note the attenuation with height and the absence of vertical phase shift. The pressure distribution, with highs in valleys and lows on hill tops, causes no drag on the moving air mass.

10-5 A NOTE ON NONLINEAR EFFECTS

All we have said thus far on internal waves is strictly applicable only if the amplitudes are small. But how small is small? The answer lies in comparing the displacements of the particles caused by the wave to the wavelength: If those displacements are much smaller than the wavelength, then advective processes are unimportant and the linear analysis is justified. The maximum horizontal displacement of fluid particles subject to an oscillatory horizontal velocity of the type $u = U \sin(lx + nz - \omega t)$ *is* U/ω, whereas the horizontal wavelength is $2\pi/l$. We thus require $U/\omega \ll 2\pi/l$, or because of (10-8),

$$U \ll \frac{2\pi N}{\sqrt{l^2 + n^2}} \leq \frac{2\pi N}{n}.$$

Since $2\pi/n$ is the vertical wavelength, which we can take as the depth scale of the motion and denote by H, the criterion becomes

$$Fr = \frac{U}{NH} \ll 1. \qquad (10\text{-}23)$$

Thus, the preceding description of internal waves is applicable only to situations where the Froude number (based on the wave-induced velocities and the vertical wavelength) is much less than unity. Note that NH is approximately the horizontal phase speed of the wave, and the criterion can be equivalently stated as a restriction to fluid velocities much smaller than the wave speed. When the preceding condition is not met, nonlinear effects cannot be neglected, and the spectral analysis fails.

A first possible effect is wave breaking. The crest (or trough) overtakes the rest of the wave, and the wave rolls over not unlike the surf on the sea surface upon approaching a beach. This type of instability, due to the wave motion itself, is termed *advective instability*. At lower energy levels, waves do not overturn but may nonetheless be sufficiently strong not to conform to the linear theory. Wave interactions create harmonics, and energy spreads over a continuous spectrum, usually spanning several octaves in frequencies and wave numbers.

Observed spectra in the deep ocean (i.e., in areas remote from important topographic influences) all show a striking resemblance (Munk, 1981), suggesting the existence of a universal spectrum. This observation led Christopher J. R. Garrett and Walter H. Munk to formulate in 1972 a prototypical spectrum for internal-wave energy. This model spectrum was subsequently modified and refined (Garrett and Munk, 1979; Munk, 1981) and has become known as the *Garrett–Munk spectrum*. It is largely empirical in the sense that its formulation is based on simple physical considerations and numerical adjustment. Yet, it has been shown to conform to a large number of observations, prompting the conjecture (Munk, 1981) that the internal-wave climate in the deep ocean is somehow regulated by a saturation process rather than by external generation processes. No theory has yet confirmed this conjecture.

In coastal areas, where topographic irregularities play a dominant role in generating internal waves, it is not unusual to find coherent wave groups at a single (tidal) frequency. Under certain conditions, the dispersion effect (different wave speeds for different wave numbers) can annihilate the nonlinear steepening effect (crests or troughs overtaking the rest of the wave), yielding a robust wave called *internal solitary wave* (Turner, 1973, Chapter 3). Figure 10-2 displays the surface signature of a train of internal solitary waves.

10-6 A NOTE ON SHEAR EFFECTS

Theory, field observations, and laboratory simulations indicate that internal-wave characteristics are substantially altered in the presence of shear flows. Although a general theory is beyond our present scope, it is worth noting that, like waves in a laterally sheared flow of a homogeneous fluid, internal waves can also encounter critical levels in a vertically sheared flow (wave speed equal to local flow velocity).

For weak to moderate shear flows ($du/dz < 2N$) in nonrotating (Booker and Bretherton, 1967) and rotating (Jones, 1967) stratified fluids, theoretical considerations show that upon approaching a critical level, the internal-wave vertical wave number increases without limit and the group velocity becomes horizontal, thus aligning itself with the flow. The theory also shows that the time taken for the energy to reach the critical level is infinite, implying that dissipative effects become important. Physically, the energy is not focused and amplified but absorbed and dissipated at a critical level.

In stronger shear flows ($du/dz > 2N$), instabilities develop. This *shear instability*, treated in the following chapter, is the vertical analogue of the barotropic instability in horizontally sheared flows (Chapter 7).

PROBLEMS

10-1. In a coastal ocean, the water density varies from 1028 kg/m^3 at the surface to 1030 kg/m^3 at a depth 100 m. What is the maximum internal-wave frequency? What is the corresponding period?

10-2. Internal waves are generated along the coast of Norway by the M_2 surface tide (period of 12.42 h). If the buoyancy frequency N is 2×10^{-3} s^{-1}, at which possible angles can the energy propagate with respect to the horizontal? (*Hint*: Energy propagates in the direction of the group velocity.)

10-3. Derive the dispersion relation of internal gravity waves in the presence of rotation, assuming $f < N$. Show that the frequency of these waves must always be higher than f but lower than N. Compare vertical phase speed to vertical group velocity.

10-4. A 10-m/s wind blows over a rugged terrain, and lee waves are generated. If the stratification frequency is equal to 0.03 s^{-1} and if the topography is approximated to a sinusoidal pattern aligned perpendicularly to the wind, with a 25-km wavelength and a height difference from trough to crest of 500 m, calculate the vertical wavelength, the angle made by the wave fronts (surfaces of constant phase) with the horizontal, and the maximum horizontal velocity at the ground. Also, where is this maximum velocity observed (at crests, at troughs, or at the points of maximum slope)?

10-5. A 75-km/h gale wind blows over a hilly countryside. If the terrain elevation is approximated by a sinusoid of wavelength 4 km and amplitude of 40 m and if the stratification frequency of the air mass is 0.025 s^{-1}, what are the vertical displacements of the air particles at 1000 m and 2000 m above the mean ground level?

SUGGESTED LABORATORY DEMONSTRATION

Equipment

A glass or plexiglass container of arbitrary shape, dyed sugar water (or sweet colored drink such as cranberry juice), water, wooden object with string.

Experiment

Fill the container halfway with the dyed sugar water (or substitute). Continue filling, delicately, with plain water. To avoid mixing and create a stratification, pour the water slowly along the sides or through a floating dish with bottom perforations. Let the fluid sit for a few minutes to allow damping of motions and to permit some diffusion across the fluid interface. Float the wooden object and move it across the container (lengthwise, if the container is elongated) by pulling slowly on the string. Observe the internal waves by looking laterally through the side of the container. (For the best effect, have both fluid layers of equal depth, each slightly deeper than the submerged portion of the wooden object.)

Walter Heinrich Munk

1917 –

Born in Austria and educated in the United States, Walter Heinrich Munk became interested in oceanography during a summer project under Harald Sverdrup at the Scripps Institution of Oceanography and quickly developed a fascination for ocean waves. This interest in waves arose partly because of the wartime need to predict sea and swell and also because Munk found wave research a challenge of intermediate complexity between simple periodic oscillations and hopeless chaos. As years went by, Munk eventually investigated all wavelengths, from the small capillary waves responsible for sun glitter to the ocean-wide tides. His studies of internal waves, in collaboration with Christopher Garrett, led him to propose a universal spectrum for the distribution of internal-wave energy in the deep ocean, now called the Garrett–Munk spectrum. More recently, pursuing an interest in acoustic waves, Munk initiated ocean tomography, a method for determining the large-scale temperature structure in the ocean from the measure of acoustic travel times. (*Photo by Jeff Cordia.*)