19

Equatorial Dynamics

Summary: Because the Coriolis force vanishes along the equator, tropical regions exhibit particular dynamics. After an overview of linear waves that exist only along the equator, the chapter concludes with a brief presentation of the episodic transfer of warm waters from the western to the eastern tropical Pacific Ocean, a phenomenon called El Niño.

19-1 EQUATORIAL BETA PLANE

Along the equator (latitude $\varphi = 0°$), the Coriolis parameter $f = 2\Omega \sin \varphi$ obviously vanishes. Without a Coriolis force, currents cannot be maintained in geostrophic balance, and we expect dramatic dynamical differences between tropical and extratropical regions. The first question is the determination of the meridional extent of the tropical region where these special effects can be expected.

It is most natural here to choose the equator as the origin of the meridional axis. The beta-plane approximation to the Coriolis parameter (see Section 6-4) then yields

$$f = \beta_0 y, \tag{19-1}$$

where y measures the meridional distance from the equator (positive northward) and

$\beta_0 = 2\Omega/a = 2.28 \times 10^{-11}\ \mathrm{m}^{-1}\cdot\mathrm{s}^{-1}$ with Ω and a being, respectively, the earth's angular rotation rate and radius ($\Omega = 7.29 \times 10^{-5}\ \mathrm{s}^{-1}$, $a = 6371$ km). This representation of the Coriolis parameter bears the name of *equatorial beta-plane* approximation.

Our previous considerations of midlatitude processes (see Chapter 15, for example) point to the important role of the internal deformation radius,

$$R = \frac{\sqrt{g'H}}{f} = \frac{c}{f}, \tag{19-2}$$

in governing the extent of dynamical structures. Here, g' is a suitable reduced gravity characterizing the stratification and H is a layer thickness. Naturally, if this distance from a given meridional position y includes the equator, equatorial dynamics must supersede midlatitude dynamics. Thus, a criterion to determine the width R_{eq} of the tropical region is

$$R_{eq} = R \quad \text{at} \quad y = R_{eq}.$$

Substituting (19-1) into (19-2), the criterion yields

$$R_{eq} = \sqrt{\frac{c}{\beta_0}}, \tag{19-3}$$

which is called the *equatorial radius of deformation*. For the previously quoted value of β_0 and for $c = (g'H)^{1/2} = 1.4$ m/s, typical of the tropical ocean (Philander, 1990, Chapter 3), we estimate $R_{eq} = 248$ km, or 2.23° of latitude. Because the stratification of the atmosphere is far stronger than that of the ocean, the equatorial radius of deformation is several times larger in the atmosphere. This implies that connections between tropical and temperate latitudes are different in the atmosphere and oceans.

Since c is a velocity (to be related shortly to a wave speed), we can define an *equatorial inertial time* T_{eq} as the travel time to cover the distance R_{eq} at speed c. Simple algebra yields

$$T_{eq} = \frac{1}{\sqrt{\beta_0 c}}, \tag{19-4}$$

which, for the previous values, is about 2 days.

19-2 LINEAR WAVE THEORY

Because of the important role they play in the so-called El Niño phenomenon, the focus of this section is on oceanic waves. The stratification of the equatorial ocean generally consists of a distinct warm layer separated from the deeper waters by a shallow thermocline (Figure 19-1). Typical values are $\Delta\rho/\rho_0 = 0.002$ and thermocline depth $H = 100$ m (leading to the previously quoted value of $c = (g'H)^{1/2} = 1.4$ m/s). This suggests the use of a one-layer reduced-gravity model, which for the purpose of a wave theory is immediately linearized:

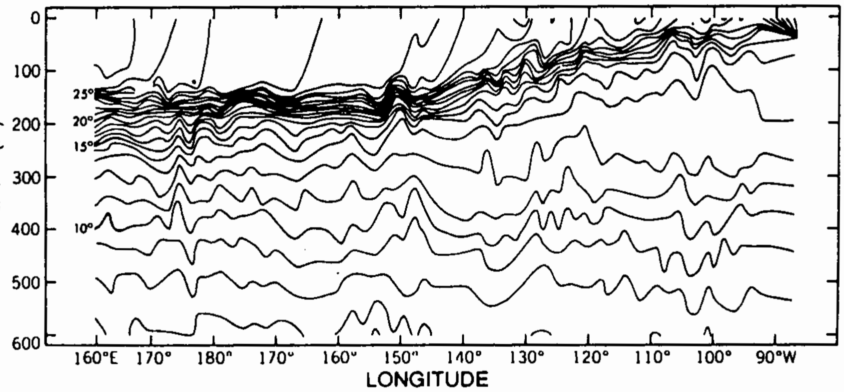

Figure 19-1 Temperature (°C) as a function of depth and longitude along the equator in the Pacific Ocean as measured in 1963 by Colin et al. (1971). Note the strong thermocline between 100 m and 200 m.

$$\frac{\partial u}{\partial t} - \beta_0 y v = -g' \frac{\partial \eta}{\partial x}, \tag{19-5a}$$

$$\frac{\partial v}{\partial t} + \beta_0 y u = -g' \frac{\partial \eta}{\partial y}, \tag{19-5b}$$

$$\frac{\partial \eta}{\partial t} + H \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = 0. \tag{19-5c}$$

Here u and v are, respectively, the zonal and meridional velocity components, g' is the reduced gravity $g\Delta\rho/\rho_0$ ($= 0.02$ m/s^2), and η is the layer-thickness variation (measured positively downward).

The preceding set of equations admits a solution with zero meridional flow. With $v = 0$, (19-5a) through (19-5c) then reduce to

$$\frac{\partial u}{\partial t} = -g' \frac{\partial \eta}{\partial x}, \quad \frac{\partial \eta}{\partial t} + H \frac{\partial u}{\partial x} = 0,$$

having any function of $x \pm ct$ and y as its solution. The remaining equation, (19-5b), sets the meridional structure, which for a signal decaying away from the equator is given by

$$u = cF(x - ct)\ e^{-y^2/2R_{eq}^2}, \tag{19-6a}$$

$$v = 0, \tag{19-6b}$$

$$\eta = HF(x - ct)\ e^{-y^2/2R_{eq}^2}, \tag{19-6c}$$

where $F(\ )$ is an arbitrary function of its argument and $R_{eq} = (c/\beta_0)^{1/2}$ is the equatorial radius of deformation introduced in the preceding section. This solution describes a wave traveling eastward at speed c, with maximum amplitude along the equator and decaying symmetrically with latitude over a distance on the order of the equatorial radius of deformation. The analogy with the coastal Kelvin wave exposed in Section 6-2 is immediate (wave speed equal to gravitational wave speed, absence of transverse flow, and decay over a deformation radius); for this reason, it is called the ***equatorial Kelvin wave***. Credit for the discovery of such a wave, however, must go to Wallace and Kousky (1968).

The set of equations (19-5) admits additional wave solutions, more akin to inertiagravity (Poincaré) and planetary (Rossby) waves. To find these, let us seek sinusoidal solutions in time and the zonal direction:

$$u = U(y)\cos(kx - \omega t),$$
$$v = V(y)\sin(kx - \omega t),$$
$$\eta = A(y)\cos(kx - \omega t).$$

Elimination of the $U(y)$ and $A(y)$ amplitude functions yields a single equation governing the meridional structure $V(y)$ of the meridional velocity:

$$\frac{d^2V}{dy^2} + \left(\frac{\omega^2 - \beta_0^2 y^2}{c^2} - \frac{\beta_0 k}{\omega} - k^2\right)V = 0. \qquad (19\text{-}7)$$

Because the expression between parentheses depends on the variable y, the solutions to this equation are not sinusoidal. In fact, for values of y sufficiently large, this coefficient becomes negative, and we anticipate exponential decay at large distances from the equator. It can be shown that solutions to (19-7) are of the type

$$V(y) = H_n\left(\frac{y}{R_{eq}}\right) e^{-y^2/2R_{eq}^2}, \qquad (19\text{-}8)$$

where H_n is a polynomial of degree n, and that these solutions exist only if

$$\frac{\omega^2}{c^2} - k^2 - \frac{\beta_0 k}{\omega} = \frac{2n+1}{R_{eq}^2}. \qquad (19\text{-}9)$$

Thus the waves form a discrete set of modes ($n = 0, 1, 2, \ldots$). Equation (19-9) is the dispersion relation providing frequencies ω as a function of wave number k for each mode. As Figure 19-2 shows, three ω roots exist for each n as k varies. The largest positive and negative roots for $n \geq 1$ correspond to frequencies greater than the equatorial inertial time. The slight asymmetry in these curves is caused by the beta term in (19-9). Without this term, the frequencies can be approximated by

$$\omega_n \simeq \pm\sqrt{\frac{2n+1}{T_{eq}} + g'H\,k^2}, \qquad n \geq 1, \qquad (19\text{-}10)$$

which is analogous to (6-14), the dispersion relation of inertia-gravity waves. These waves are thus the low-latitude extensions of the inertia-gravity waves (Section 6-3).

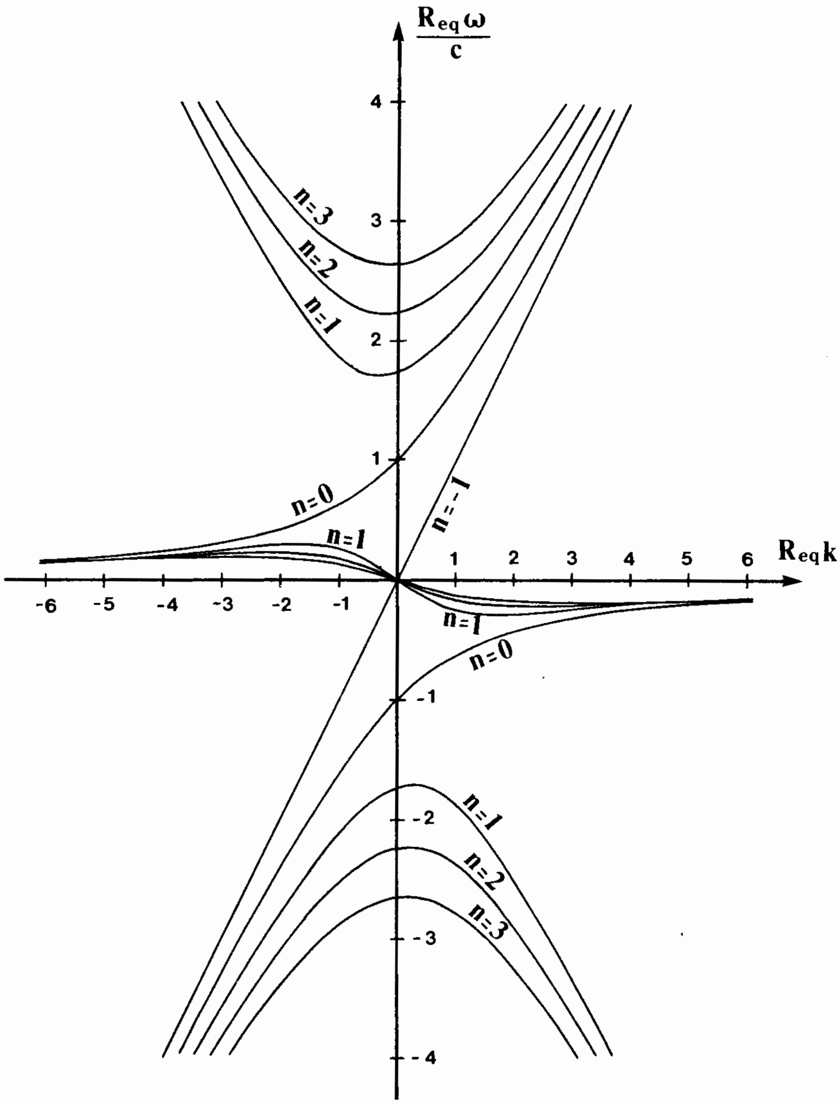

Figure 19-2 Dispersion diagram for equatorially trapped waves.

The third and much smaller roots for $n \geq 1$ correspond to subinertial frequencies and thus to tropical extensions of the midlatitude planetary waves (Section 6-4). At long wavelengths (small k values), these waves are nearly nondispersive and propagate westward at the speeds

$$c_n = \frac{\omega_n}{k} \simeq -\frac{\beta_0 R_{eq}^2}{2n+1}, \quad n \geq 1, \tag{19-11}$$

which are to be compared with (6-25). The case $n = 0$ has the frequency given by

$$\omega T_{eq} - \frac{1}{\omega T_{eq}} = kR_{eq}, \tag{19-12}$$

and, as Figure 19-2 shows, exhibits a mixed behavior between planetary and inertia-gravity waves. Finally, the Kelvin-wave solution can be included in the set by taking $n = -1$ (Figure 19-2).

The polynomials of (19-8) are not arbitrary but must be the so-called Hermite polynomials (Abramowitz and Stegun, 1972, Chapter 22). The first few polynomials of this series are $H_0(\xi) = 1$, $H_1(\xi) = 2\xi$, and $H_2(\xi) = 4\xi^2 - 2$. Waves of even order are antisymmetric about the equator $[\eta(-y) = -\eta(y)]$, whereas those of odd order are symmetric $[\eta(-y) = \eta(y)]$. Thus, the mixed wave is antisymmetric and the Kelvin wave is symmetric.

When the equatorial ocean is perturbed (e.g., by changing winds), its adjustment toward a new state is accomplished by wave propagation. At low frequencies (periods

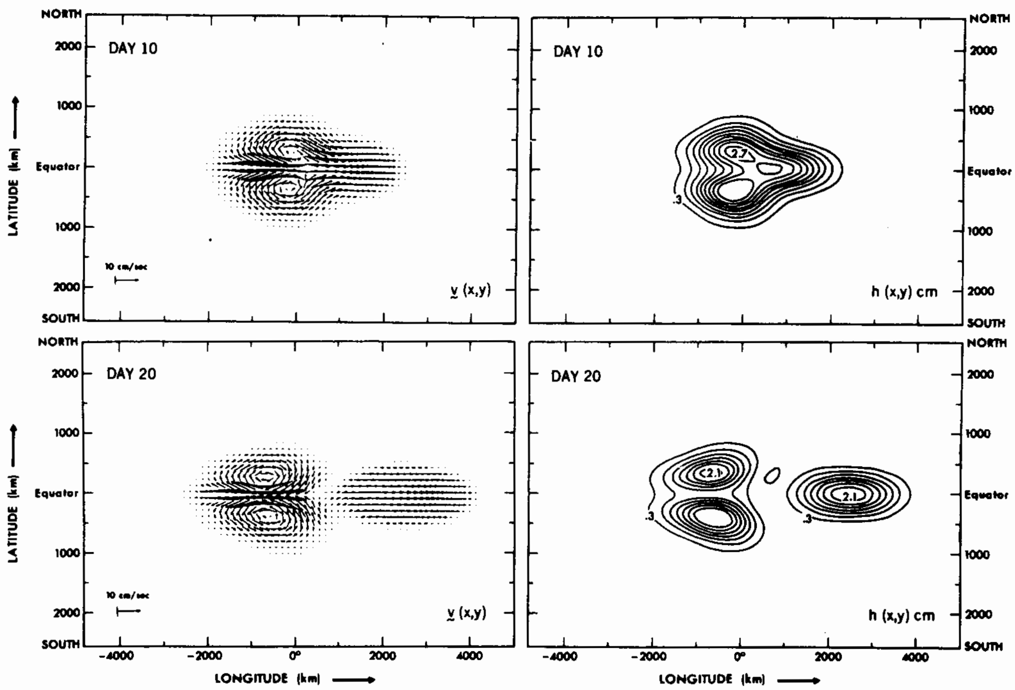

Figure 19-3 The dispersion of an initial bell-shaped downward displacement of the interface imposed on a stretch of equatorial ocean. Clearly visible are the one-bulge Kelvin wave propagating eastward and a double-bulge planetary (Rossby) wave going westward. (From Philander et al., 1984.)

longer than T_{eq}, or about 2 days), inertia-gravity waves are not excited, and the ocean's response consists entirely of the Kelvin wave, the mixed wave, and some planetary waves (those of appropriate frequencies). If, moreover, the perturbation is symmetric about the equator (and generally there is a high degree of symmetry about the equator), the mixed wave and all planetary waves of even order are ruled out. The Kelvin wave and odd planetary waves of short wavelengths (if any) carry energy eastward, whereas the odd planetary waves of long wavelengths carry energy westward. Figure 19-3 displays the temporal dispersion of a bell-shaped thermocline displacement imposed on a stretch of equatorial ocean. Clearly visible are the one-bulge Kelvin wave progressing eastward and the double-bulge lowest planetary wave ($n = 1$) propagating westward. Although this case is obviously academic, it is believed that Kelvin waves and low-order planetary waves, together with wind-driven currents, are prevalent in the equatorial ocean.

At this point, a number of interesting topics can be presented, such as the reflection of a Kelvin wave upon encountering an eastern boundary, waves around islands, and the generation of equatorial currents by time-dependent winds. But, we shall leave these matters for the more specialized literature (Gill, 1982; Philander, 1990; McPhaden and Ripa, 1990; and references therein) and limit ourselves to the presentation of the El Niño phenomenon.

19-3 EL NIÑO

Every year, around the Christmas season, warm waters flow along the western coast of South America from the equator to Peru and beyond. These waters, which are several degrees warmer than usual and are much less saline, perturb the coastal ocean, suppressing—among other things—the semipermanent coastal upwelling of cold waters. So noticeable is this phenomenon that is has become known as El Niño, which in Spanish means "the child" or more specifically the Christ Child, in relation to the Christmas season.

Regularly but not periodically (every 2 to 4 years), the amount of passing warm waters is substantially greater than in normal years, and life in those regions is greatly perturbed, for better and for worse. Anomalously abundant precipitations, caused by the warm ocean, can in a few weeks turn the otherwise arid coastal region of Peru into a land of plenty. But, suppression of coastal upwelling causes widespread destruction of plankton and fish. The ecological and economic consequences are noticeable. In Peru, the fish harvest is much reduced, sea birds (which prey on fish) die in large numbers, and, to compound the problem, dead fish and birds rotting on the beach create unsanitary atmospheric conditions.

In the scientific community, the name El Niño is being restricted to such anomalous occurrences. Major El Niño events of this century occurred in 1925, 1941, 1957–1958, 1972–1973, 1982–1983, and 1992. Their cause remained obscure until Wyrtki (1973) discovered a strong correlation with changes in the central and western tropical Pacific Ocean, thousands of kilometers away. It is now well established (Philander, 1990) that

El Niño events are caused by changes in the surface winds over the tropical Pacific, which episodically release and drive warm waters, previously piled by the trade winds in the western half of the basin, eastward to the American continent and southward along the coast. The situation is quite complex, and it took oceanographers and meteorologists more than a decade to understand the various oceanic and atmospheric factors.

Under normal conditions, winds over the tropical Pacific Ocean consist of the northeast trade winds (northeasterlies) and the southeast trade winds (southeasterlies) that converge over the *intertropical convergence zone* (ITCZ) and blow westward (Section 18-3). Although it migrates meridionally in the course of the year, the ITCZ sits predominantly in the Northern Hemisphere (around 5° to 10°N). In addition to accumulating a great amount of warm water in the western tropical Pacific, the trades also generate equatorial upwelling (Section 14-1) over the eastern part of the basin. Thus, in a normal situation, the western tropical Pacific has a warm-water pool, whereas its eastern portion is relatively cold. (This characteristic is evidenced by the westward deepening of the thermocline, as shown in Figure 19-1.)

The origin of an anomalous, El Niño event is associated with a weakening of the trade winds in the western Pacific or with the appearance of a warm sea-surface-temperature (SST) anomaly in the central tropical Pacific. Although one may precede the other, they soon go hand in hand. A slackening of the western trades relaxes the thermocline slope and releases some of the warm waters; this relaxation takes the form of a downwelling Kelvin wave, whose wake is thus a warm SST anomaly. On the other hand, a warm SST anomaly locally heats the atmosphere, creating ascending motions that need to be compensated by horizontal convergence. This horizontal convergence naturally calls for eastward winds on its western side, thus weakening or reversing the trade winds there (Gill, 1980). In sum, a relaxation of the trade winds in the western Pacific creates a warm sea-surface anomaly, and vice versa. Feedback occurs and the perturbation amplifies. On the eastern side of the anomaly, convergence calls for a strengthening of the trades that, in turn, enhances equatorial upwelling. This cooling interferes with the eastward progression of the downwelling Kelvin wave, and it is not clear which should dominate. During an El Niño event, the anomaly does travel eastward while amplifying. Once the warm water arrives at the American continent, it separates into a weaker northward branch and a stronger southward branch, each becoming a coastal Kelvin wave (downwelling). The subsequent events are as described at the beginning of this section.

When an El Niño event occurs, its temporal development is strictly controlled by the annual cycle. The warm waters arrive in Peru around December, and the seasonal variation of the general atmospheric circulation calls for a northward return of the ITCZ and a re-establishment of the southeast trade winds along the equator. The situation returns to normal.

This sequence of events is now well understood (Philander, 1990) and has been successfully modeled (Cane et al., 1986). Models are now routinely used to forecast the next occurrence of an El Niño event and its intensity with a lead time of 6 to 9 months. What remains less clear is the variability of the atmosphere-ocean system on the scales of several years. A connection with the called *Southern Oscillation* has been made clear, and the phenomenon is sometimes called ENSO, for El Niño–Southern Oscillation. The Southern Oscillation is a quasi-periodic variation of the surface atmospheric pressure distribution over large portions of the globe. In particular, it has been observed that pressure changes at Darwin (in northern Australia, at 12°S, 131°E) are almost perfectly negatively corrolated with pressure variations at the island of Tahiti (18°S, 149°W). The presence of high monthly averaged pressures in Darwin with simultaneous low pressures in Tahiti is intimately connected with each El Niño occurrence. Explaining this connection evidently demands a deep understanding of the natural variations and instabilities of the coupled tropical-ocean–global-atmosphere system. This topic is the subject of intense research at the present time.

PROBLEMS

19-1. How long does an equatorial Kelvin wave take to cross the entire Pacific Ocean?

19-2. Generalize the equatorial-Kelvin-wave theory to the uniformly stratified ocean. Assume inviscid and nonhydrostatic motions. Discuss analogies with internal waves.

19-3. Show that equatorial upwelling (mentioned in Section 14-1; see Figure 14-2) must be confined at low frequencies to a width on the order of the equatorial radius of deformation.

19-4. In the Indian Ocean, two current-meter moorings placed at the same longitude and symmetrically about the equator (1.5° of latitude) record velocity oscillations with a dominant period of 12 days. Furthermore, the zonal velocity at the northern mooring lags by a quarter of a period the meridional velocities of both moorings and by half a period the zonal velocity at the southern mooring. The stratification provides $c = 1.2$ m/s. What kind of wave is being observed? What is its zonal wavelength? Can a comparison of the maximum zonal and meridional velocities provide a confirmation of this wavelength?

19-5. Consider geostrophic adjustment in the tropical ocean. What would be the final steady state following the release of buoyant waters with zero potential vorticity along the equator of an infinitely deep and motionless ocean? For simplicity, assume zonal invariance and equatorial symmetry.

Adrian Edmund Gill

1937 – 1986

Born in Australia, Adrien Edmund Gill pursued his career in Great Britain. His publications spanned a wide range of topics, including wind-forced currents, equatorially trapped ocean waves, tropical atmospheric circulation, and the El Niño–Southern Oscillation phenomenon, and culminated in his treatise *Atmosphere-Ocean Dynamics* (Academic Press, 1982). His greatest contributions relied on the formulation of simple yet illuminating models of geophysical flows. It has been said (only half jokingly) that he could reduce all problems to a simple ordinary differential equation with constant coefficients, with all the essential physics retained. Although he never held a professorship, Gill supervised numerous students at the Universities of Cambridge and Oxford. He is also remembered for his unassuming style and for the generosity with which he shared his ideas with students and colleagues. (*Photo credit: Gillman and Soame, Oxford.*)