# 2

# The Coriolis Force

**Summary:** The object of this chapter is to examine the Coriolis force, a fictitious force arising from the choice of a rotating framework of reference. Some physical considerations are offered to provide insight on this nonintuitive but essential element of geophysical flows.

## *2-1 MOTIVATION FOR THE CHOICE OF A ROTATING REFERENCE FRAMEWORK*

From a theoretical point of view, all equations governing geophysical fluid processes could be stated with respect to an inertial framework of reference, fixed with respect to distant stars. However, we people on Earth observe fluid motions with respect to this rotating system. Also, mountains and ocean boundaries are stationary with respect to Earth. Common sense dictates that we write the governing equations in a reference framework rotating with our planet. (The same can be said for other planets and stars.) The trouble arising from the additional terms in the equations of motion is less than that which would arise from having to reckon with moving boundaries and the need to subtract systematically the ambient rotation from the results.

## 2-2 ROTATING FRAME OF REFERENCE

To facilitate the mathematical developments, let us first investigate the two-dimensional case (Figure 2-1). Let the $X$- and $Y$-axes form the inertial framework of reference and the $x$- and $y$-axes be those of a framework with the same origin but rotating at the

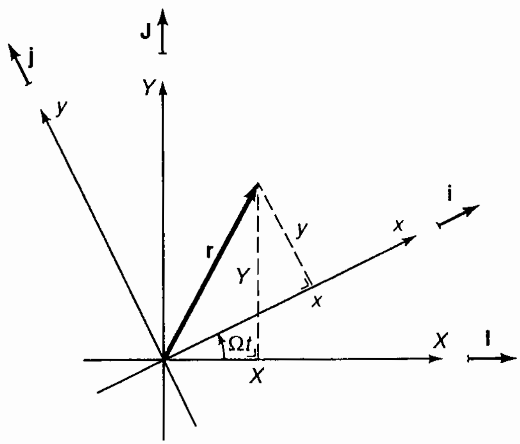


**Figure 2-1** Fixed $(X, Y)$ and rotating $(x, y)$ frameworks of reference.

angular rate $\Omega$ (defined as positive in the trigonometric sense). The corresponding unit vectors are denoted $(\mathbf{I}, \mathbf{J})$ and $(\mathbf{i}, \mathbf{j})$. At any time $t$, the rotating $x$-axis makes an angle $\Omega t$ with the fixed $X$-axis. It follows that

$$\mathbf{i} = \mathbf{I} \cos \Omega t + \mathbf{J} \sin \Omega t, \tag{2-1a}$$

$$\mathbf{j} = -\mathbf{I} \sin \Omega t + \mathbf{J} \cos \Omega t \tag{2-1b}$$

and that the coordinates of the position vector $\mathbf{r} = X\,\mathbf{I} + Y\,\mathbf{J} = x\,\mathbf{i} + y\,\mathbf{j}$ of any point in the plane are related by

$$x = X \cos \Omega t + Y \sin \Omega t, \tag{2-2a}$$

$$y = -X \sin \Omega t + Y \cos \Omega t. \tag{2-2b}$$

The first time derivative of the preceding expressions yields

$$\frac{dx}{dt} = \frac{dX}{dt} \cos \Omega t + \frac{dY}{dt} \sin \Omega t - \Omega X \sin \Omega t + \Omega Y \cos \Omega t \tag{2-3a}$$

$$\frac{dy}{dt} = -\frac{dX}{dt} \sin \Omega t + \frac{dY}{dt} \cos \Omega t - \Omega X \cos \Omega t - \Omega Y \sin \Omega t. \tag{2-3b}$$

The quantities $dx/dt$ and $dy/dt$ give the rates of change of the coordinates relative to the moving frame as time evolves. They are thus the components of the relative velocity:

$$\mathbf{u} = \frac{dx}{dt}\,\mathbf{i} + \frac{dy}{dt}\,\mathbf{j} = u\mathbf{i} + v\mathbf{j}. \tag{2-4}$$

Similarly, $dX/dt$ and $dY/dt$ give the rates of change of the absolute coordinates and form the absolute velocity:

$$\mathbf{U} = \frac{dX}{dt}\,\mathbf{I} + \frac{dY}{dt}\,\mathbf{J}.$$

Writing the absolute velocity in terms of the rotating unit vectors, we obtain [using (2-1)]

$$\mathbf{U} = \left(\frac{dX}{dt}\cos\Omega t + \frac{dY}{dt}\sin\Omega t\right)\mathbf{i} + \left(-\frac{dX}{dt}\sin\Omega t + \frac{dY}{dt}\cos\Omega t\right)\mathbf{j}$$
$$= U\mathbf{i} + V\mathbf{j}. \tag{2-5}$$

Thus, $dX/dt$ and $dY/dt$ are the components of the absolute velocity $\mathbf{U}$ in the inertial frame, whereas $U$ and $V$ are the components of the same vector in the rotating frame. Use of (2-3) and (2-2) in the preceding expression yields the following relations between absolute and relative velocities:

$$U = u - \Omega y, \qquad V = v + \Omega x. \tag{2-6}$$

These equalities simply state that the absolute velocity is the relative velocity plus the entraining velocity due to the rotation of the reference framework.

A second derivative with respect to time provides in a similar manner:

$$\frac{d^2x}{dt^2} = \left(\frac{d^2X}{dt^2}\cos\Omega t + \frac{d^2Y}{dt^2}\sin\Omega t\right) + 2\Omega\left(-\frac{dX}{dt}\sin\Omega t + \frac{dY}{dt}\cos\Omega t\right)$$
$$- \Omega^2(X\cos\Omega t + Y\sin\Omega t) \tag{2-7a}$$

$$\frac{d^2y}{dt^2} = \left(-\frac{d^2X}{dt^2}\sin\Omega t + \frac{d^2Y}{dt^2}\cos\Omega t\right) - 2\Omega\left(\frac{dX}{dt}\cos\Omega t + \frac{dY}{dt}\sin\Omega t\right)$$
$$- \Omega^2(-X\sin\Omega t + Y\cos\Omega t). \tag{2-7b}$$

Expressed in terms of the relative and absolute accelerations

$$\mathbf{a} = \frac{d^2x}{dt^2}\,\mathbf{i} + \frac{d^2y}{dt^2}\,\mathbf{j} = \frac{du}{dt}\,\mathbf{i} + \frac{dv}{dt}\,\mathbf{j} = a\mathbf{i} + b\mathbf{j}$$

$$\mathbf{A} = \frac{d^2X}{dt^2}\,\mathbf{I} + \frac{d^2Y}{dt^2}\,\mathbf{J} = A\mathbf{i} + B\mathbf{j},$$

the last expressions condense to

$$a = A + 2\Omega V - \Omega^2 x, \qquad b = B - 2\Omega U - \Omega^2 y.$$

In analogy with the absolute velocity vector, $d^2X/dt^2$ and $d^2Y/dt^2$ are the components of the absolute acceleration $\mathbf{A}$ in the inertial frame, whereas $A$ and $B$ are the components of the same vector in the rotating frame. The absolute acceleration components, necessary later to formulate Newton's law, are obtained by solving for $A$ and $B$:

$$A = a - 2\Omega v - \Omega^2 x, \qquad B = b + 2\Omega u - \Omega^2 y. \tag{2-8}$$

We now see that the difference between absolute and relative acceleration consists of two contributions. The first, proportional to $\Omega$ and to the velocity, is called the Coriolis acceleration; the other, proportional to $\Omega^2$ and to the coordinates, is called the centripetal acceleration. When placed on the other side of the equality in Newton's law, these terms can be assimilated to forces. The centrifugal force acts as an outward pull, whereas the Coriolis force depends on the direction and magnitude of the relative velocity.

Formally, the preceding results could have been derived in a vector form. Defining the vector rotation

$$\mathbf{\Omega} = \Omega \mathbf{k},$$

where $\mathbf{k}$ is the unit vector in the third dimension (which is common to both systems of reference), we can write (2-6) and (2-8) as

$$\begin{aligned} \mathbf{U} &= \mathbf{u} + \mathbf{\Omega} \times \mathbf{r} \\ \mathbf{A} &= \mathbf{a} + 2\mathbf{\Omega} \times \mathbf{u} + \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}), \end{aligned} \tag{2-9}$$

which implies that taking a time derivative of a vector with respect to the inertial framework is equivalent to applying the operator

$$\frac{d}{dt} + \mathbf{\Omega} \times$$

in the rotating framework of reference.

A very detailed exposition of the Coriolis and centripetal accelerations can be found in the book by Stommel and Moore (1989).

## 2-3 UNIMPORTANCE OF THE CENTRIFUGAL FORCE

Unlike the Coriolis force, which is proportional to the velocity, the centrifugal force depends solely on the rotation rate and the distance of the particle to the rotation axis. Even at rest with respect to the rotating planet, particles experience an outward pull. Yet, on the earth as on other celestial bodies, no matter goes flying out to space. How is that possible? Obviously, gravity keeps everything together.

In the absence of rotation, gravitational forces keep the matter together to form a spherical body (with the denser materials to the center and the lighter ones on the periphery). The outward pull caused by the centrifugal force distorts this spherical equilibrium, and the planet assumes a slightly flattened shape. The degree of flattening is precisely that necessary to keep the planet in equilibrium for its rotation rate.

The situation is depicted on Figure 2-2. By its nature, the centrifugal force is

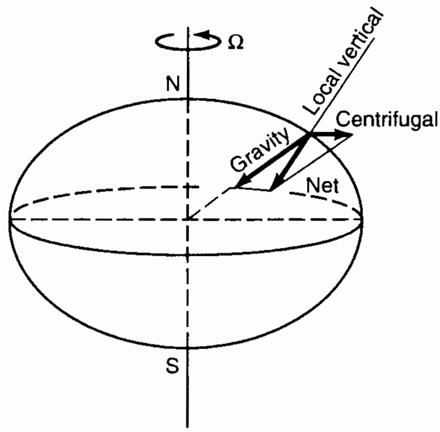


**Figure 2-2** How the flattening of the rotating earth (grossly exaggerated in this drawing) causes the gravitational and centrifugal forces to combine into a net force aligned with the local vertical, so that equilibrium is reached.

directed outward, perpendicular to the axis of rotation, whereas the gravitational force points toward the planet's center. The resulting force assumes an intermediate direction, and this direction is precisely the direction of the local vertical. Indeed, under this condition a loose particle would have no tendency of its own to fly away from the planet. In other words, every particle at rest on the surface will remain at rest unless it is subjected to additional forces.

The flattening of the earth, as well as that of other celestial bodies in rotation, is important to neutralize the centrifugal force. But, this is not to say that it greatly distorts the geometry. On the earth, for example, the distortion is very slight, because gravity by far exceeds the centrifugal force; the terrestrial equatorial radius is 6378 km, slightly greater than its polar radius of 6357 km. The shape of the rotating oblate earth is treated in detail by Stommel and Moore (1989).

For the sake of simplicity in all that follows, we will call the gravitational force the resultant force, aligned with the vertical and equal to the sum of the true gravitational force and the centrifugal force.

In a rotating laboratory tank, the situation is similar but not identical. The rotation causes a displacement of the fluid toward the periphery. This proceeds until the resulting

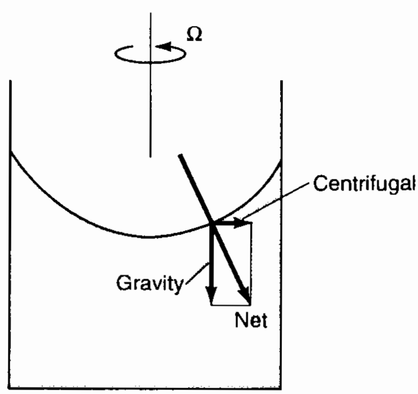


**Figure 2-3** Equilibrium surface of a rotating fluid in an open container. The surface slope is such that gravitational and local centrifugal forces combine into a net force everywhere aligned with the local normal to the surface.

inward pressure gradient prevents any further displacement. Equilibrium then requires that at any point on the surface, the downward gravitational force and the outward centrifugal force combine into a resultant force normal to the surface (Figure 2-3). Although the surface curvature is crucial in neutralizing the centrifugal force, the vertical displacements are rather small. In a tank rotating at the rate of one revolution every two seconds (30 rpm) and 40 cm in diameter, the difference in fluid height between the rim and the center is a modest 2 cm.

## 2-4 MOTION OF A FREE PARTICLE ON A ROTATING PLANE

The preceding argument allows us to combine the centrifugal force with the gravitational force, but the Coriolis force remains. To have an idea of what this force can cause, let us examine the motion of a free particle (that is, a particle not subject to any external force) on a horizontal rotating plane.

If the particle is free of any force, its acceleration in the inertial frame is nil, by Newton's law. According to (2-8), with the centrifugal-acceleration terms no longer present, the equations governing the velocity components of the particle are

$$\frac{du}{dt} - 2\Omega v = 0, \qquad \frac{dv}{dt} + 2\Omega u = 0. \tag{2-10}$$

The general solution to this system of linear equations is

$$u = V\sin(ft+\phi), \qquad v = V\cos(ft+\phi), \tag{2-11}$$

where $f = 2\Omega$, called the Coriolis parameter, has been introduced for convenience and $V$ and $\phi$ are two arbitrary constants of integration. Without loss of generality, $V$ can always be chosen as nonnegative. (Do not confuse this constant $V$ with the $y$-component of the absolute velocity introduced in Section 2-2.) A first result is that the particle speed $(u^2 + v^2)^{1/2}$ remains unchanged in time. It is equal to $V$, a constant determined by the initial conditions.

Although the speed remains unchanged, the components $u$ and $v$ do depend on time, implying a change in direction. To document this curving effect, it is most instructive to derive the trajectory of the particle. The coordinates of the particle position change, by definition of the vector velocity, according to $dx/dt = u$ and $dy/dt = v$, and a second time integration provides

$$x = x_0 - \frac{V}{f}\cos(ft+\phi) \tag{2-12a}$$

$$y = y_0 + \frac{V}{f}\sin(ft+\phi), \tag{2-12b}$$

where $x_0$ and $y_0$ are additional constants of integration to be determined from the initial coordinates of the particle. From the last relations, it follows directly that

$$(x - x_0)^2 + (y - y_0)^2 = \left(\frac{V}{f}\right)^2. \tag{2-13}$$

This implies that the trajectory is a circle centered at $(x_0, y_0)$ and of radius $V/|f|$. The situation is depicted on Figure 2-4.

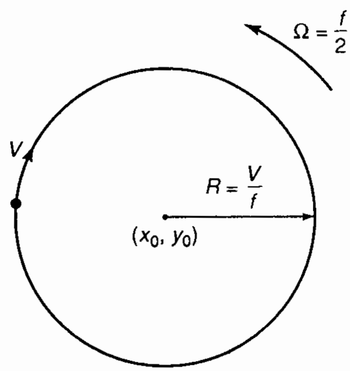


**Figure 2-4** Inertial oscillation of a free particle on a rotating plane. The orbital period is exactly half of the ambient revolution period. This figure has been drawn with a positive Coriolis parameter, $f$, representative of the Northern Hemisphere. If $f$ were negative (as in the Southern Hemisphere), the particle would veer to the left.

In the absence of rotation ($f = 0$), this radius is infinite, and the particle follows a straight path, as we could have anticipated. But, in the presence of rotation ($f \neq 0$), the particle turns constantly. A quick examination of (2-12) reveals that the particle turns to the right (clockwise) if $f$ is positive or to the left (counterclockwise) if $f$ is negative. In sum, the rule is that the particle turns in the sense opposite to that of the ambient rotation.

At this point, we may wonder whether this particle rotation is none other than the negative of the ambient rotation, in such a way as to keep the particle at rest in the absolute frame of reference. But, there are at least two reasons why this is not so. The first is that the coordinates of the center of the particle's circular path are arbitrary and are therefore not required to coincide with those of the axis of rotation. The second and most compelling reason is that the two frequencies of rotation are simply not the same: the ambient rotating plane completes one revolution in a time equal to $T_a = 2\pi/\Omega$, whereas the particle covers a full circle in a time equal to $T_p = 2\pi/f = \pi/\Omega$, called *inertial period*. Thus, the particle goes around its orbit twice as the plane accomplishes a single revolution.

The spontaneous circling of a free particle endowed with an initial velocity in a rotating environment bears the name of *inertial oscillation*. Note that, since the particle speed can vary, so can the inertial radius, $V/|f|$, whereas the frequency, $|f| = 2|\Omega|$, is a property of the rotating environment and is independent of the initial conditions.

The preceding exercise may appear rather mathematical and devoid of any physical interpretation. As far as the author is aware, nobody has yet proposed a clear and convincing physical interpretation for the Coriolis force. There exist, however, a geometric argument and a physical analogy. Let us first discuss the geometric argument.

Consider a rotating table and, on it, a particle initially ($t = 0$) at a distance $R$ from the axis of rotation, approaching the latter at a speed $u$ (Figure 2-5). At some later time $t$, the particle has approached the axis of rotation by a distance $ut$ and has covered the distance $\Omega Rt$ laterally. It now lies at the position indicated by a solid dot. During the

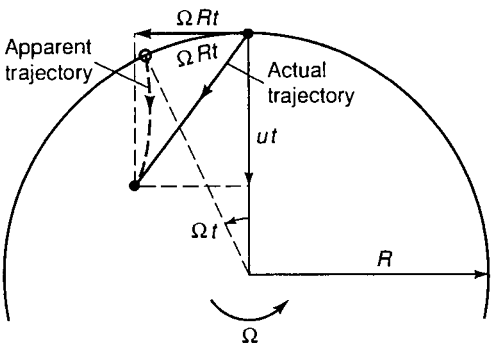


**Figure 2-5** Geometrical interpretation of the apparent veering of a particle trajectory viewed in a rotating framework. The veering is to the right if the ambient rotation is counterclockwise, as shown here.

lapse $t$, the table has rotated by an angle $\Omega t$, and, to an observer rotating with the table, the particle seems to have originated from the point on the rim indicated by the open circle. The construction shows that, although the actual trajectory is perfectly straight, the apparent path; as noted by the observer rotating with the table curves to the right.

The problem with this argument is that to construct the absolute trajectory, we chose a straight path; that is, we implicitly considered the total absolute acceleration, which in the rotating framework includes the centrifugal acceleration. The latter, however, should not have been retained, but because it is a radial force, it does not account for the transverse displacement. Therefore, the apparent veering is, at least for a short interval of time, entirely due to the Coriolis effect.

## 2-5 ANALOGY WITH A PENDULUM[1]

Consider a classical pendulum (i.e., not a rotating, Foucault pendulum) of mass $M$ and length $L$ in the gravitational field $g$ (Figure 2-6); let us review its small-amplitude oscillations. For a small angular displacement $\alpha$, the component of the gravitational force $Mg$ not balanced by the tension $T$ in the string is $F = Mg \sin \alpha \simeq Mg\alpha$ (Figure 2-6a). Because $\alpha$ is small, this force is almost horizontal. In terms of the radial distance projected on the horizontal plane, $r = L \sin \alpha \simeq \alpha L$, this force is $(g/L)r$ per unit mass. A projection onto the two Cartesian axes of the horizontal plane (Figure 2-6b) yields the following equations of motion, in an inertial frame of reference:

[1] This analogy was suggested to the author by Prof. Satoshi Sakai at Kyoto University.

$$\frac{d^2x}{dt^2} = -\frac{gr}{L}\cos\theta = -\Omega^2 x \tag{2-14a}$$

$$\frac{d^2y}{dt^2} = -\frac{gr}{L}\sin\theta = -\Omega^2 y, \tag{2-14b}$$

where $\Omega = (g/L)^{1/2}$ is the pendulum's natural frequency of oscillation at small amplitudes. Note how the gravitational restoring force takes on the form of a negative

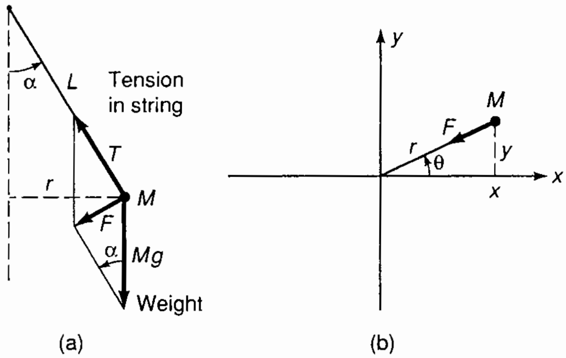


**Figure 2-6** Side view (a) and top view (b) of a classical pendulum. For small-amplitude oscillations, the acting component of the gravitational force is horizontal, directed inward, and proportional to the distance to the origin. It therefore acts as a negative centrifugal force.

centrifugal force. In polar coordinates ($x = r\cos\theta$, $y = r\sin\theta$), the preceding equations of motion become

$$\text{Radial direction:} \qquad \frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = -\Omega^2 r \tag{2-15a}$$

$$\text{Azimuthal direction:} \qquad \frac{d}{dt}\left(r^2\frac{d\theta}{dt}\right) = 0. \tag{2-15b}$$

Two particular solutions are noteworthy. If the initial pendulum perturbation is a pure radial displacement, the pendulum forever oscillates in a vertical plane of fixed azimuth $\theta$, according to

$$\frac{d^2r}{dt^2} = -\Omega^2 r. \tag{2-16}$$

The oscillation, of period $2\pi/\Omega$, takes the pendulum to the center ($r = 0$) twice per period—that is, every $\pi/\Omega$ time interval. At the other extreme, the pendulum can be imparted an initial radial displacement accompanied by an initial azimuthal velocity of magnitude such that the outward centrifugal force of the ensuing circling motion exactly cancels the inward gravitational pull at that radial distance. The pendulum then remains at a fixed distance $r$ from the center and circles at a constant rate obtained from $-r(d\theta/dt)^2 = -\Omega^2 r$; that is,

$$\frac{d\theta}{dt} = \pm\Omega. \tag{2-17}$$

The sign is in relation with the direction of the initial azimuthal velocity and the trigonometric convention for the measure of $\theta$. Outside of these two extreme behaviors, the pendulum describes elliptical trajectories of size, eccentricity, and phase related to the initial perturbation. The orbit does not take the pendulum through the center but brings it, twice per period, to a distance of closest approach (perigee) and, twice, to a distance of largest excursion (apogee).

At this point, the reader may rightfully wonder, Where is the analogy with the motion of a particle subject to the Coriolis force? To show this analogy, let us now view the pendulum motions in a rotating frame, but, of course, not any rotating frame: Let us select the angular rotation rate $\Omega$ equal to the pendulum's natural frequency. This choice is made so that, in the rotating frame of reference, the outward centrifugal force, $\Omega^2 r$, is everywhere and at all times exactly canceled by the inward gravitational force, $-\Omega^2 r$, of the pendulum. Thus, the equations of motion of the pendulum expressed in the rotating frame include only the relative acceleration and the Coriolis force, that is, they are none other than (2-10).

Let us now consider the pendulum's oscillations as seen by an observer in the rotating frame. When the pendulum oscillates strictly back and forth, the rotating observer sees a curved trajectory. Because the pendulum passes by the origin twice per oscillation, the orbit seen by the rotating observer also passes by the origin twice per period. When the pendulum reaches its extreme displacement on one side, it reaches an apogee on its orbit as viewed in the rotating frame; then, by the time it reaches its maximum displacement on the other side, $\pi/\Omega$ later, the rotating framework has rotated exactly by half a turn, so that this second apogee of the orbit falls exactly onto the first. Therefore, the reader can readily be convinced that the orbit in the rotating frame is drawn twice per period of oscillation. Algebraic or geometric developments reveal that the orbit is circular (Figure 2-7a).

In the other extreme situation, when the pendulum circles at a constant distance from the origin, two cases must be distinguished, depending on whether the pendulum circles in the direction of or opposite to the observer's rotating frame. If the direction is the same [positive sign in (2-17)], the observer simply chases the pendulum, which then appears stationary, and the orbit reduces to a single point (Figure 2-7b). This case corresponds to the state of rest of a particle in a rotating environment [$V = 0$ in (2-11) through (2-12)]. If the sense of rotation is opposite [minus sign in (2-17)], the reference frame rotates at the rate $\Omega$ in one direction, whereas the pendulum circles at the same rate in the opposite direction. To the observer, the pendulum appears to rotate at the rate $2\Omega$. The orbit is obviously a circle centered at the origin and of radius equal to the pendulum's radial displacement; it is covered twice per revolution of the rotating frame (Figure 2-7c). Finally, for arbitrary pendulum oscillations, the orbit in the rotating frame is a circle of finite radius that is not centered at the origin, does not pass by the origin, and may or may not include the origin (Figure 2-7d).

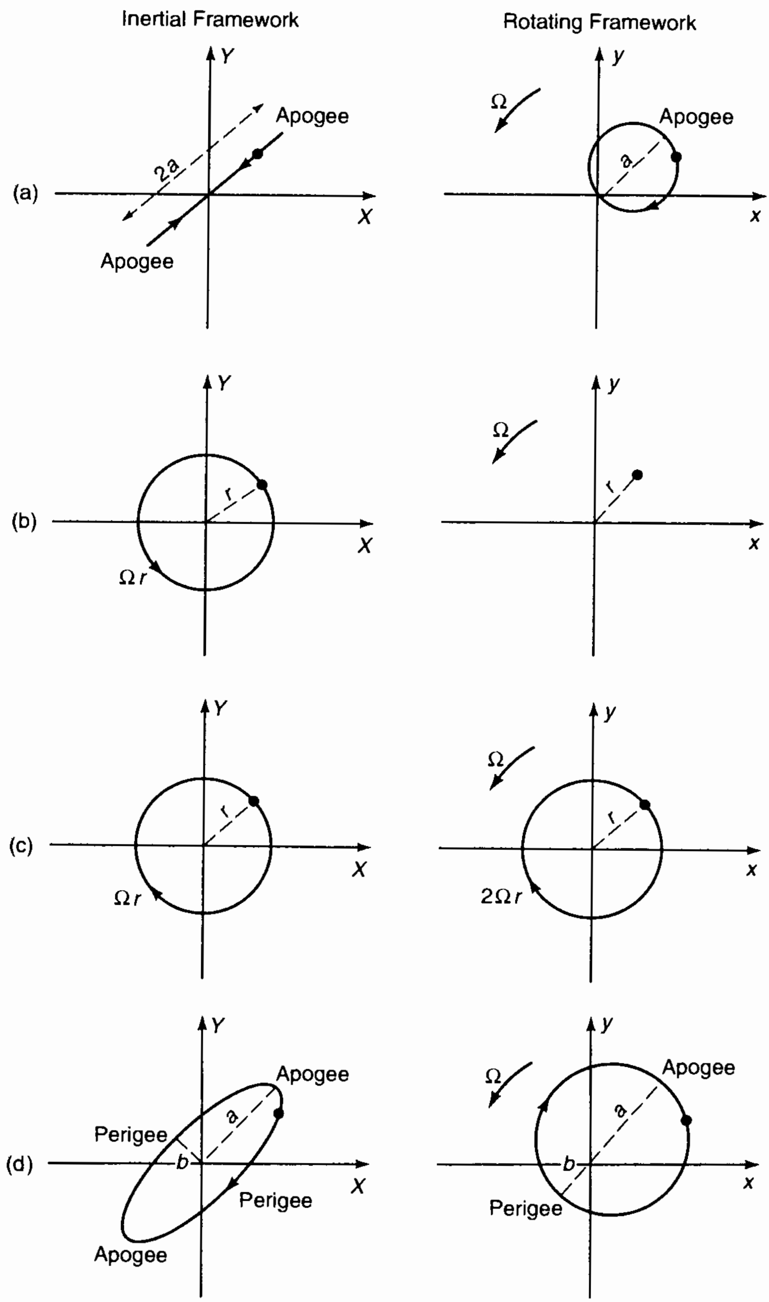


**Figure 2-7** Comparison of pendulum orbits in inertial and rotating frames of reference. Note the analogy between the orbits in the rotating frame and the trajectory of a free particle subject to a Coriolis force (Figure 2-4).

## 2-6 ACCELERATION ON A THREE-DIMENSIONAL ROTATING EARTH

For all practical purposes, except as outlined earlier when the centrifugal force was discussed (Section 2-3), the earth can be taken as a perfect sphere. This sphere rotates about its North Pole–South Pole axis. At any given latitude $\varphi$, the north–south direction departs from the local vertical, and the Coriolis force assumes a form different from that established in the preceding section.

Figure 2-8 depicts the traditional choice for a local Cartesian framework of refer-

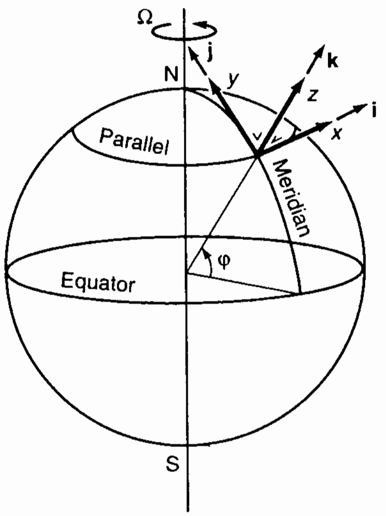


**Figure 2-8** Definition of the local Cartesian framework of reference on a spherical earth. The coordinate $x$ is directed eastward, $y$ is northward, and $z$ is upward.

ence: The $x$-axis is oriented eastward, the $y$-axis, northward, and the $z$-axis, upward. In this framework, the earth's rotation vector is expressed as

$$\mathbf{\Omega} = \Omega \cos \varphi \, \mathbf{j} + \Omega \sin \varphi \, \mathbf{k} . \tag{2-18}$$

The absolute acceleration minus the centrifugal component,

$$\frac{d\mathbf{u}}{dt} + 2\,\mathbf{\Omega} \times \mathbf{u} ,$$

has the following three components:[2]

$$x: \quad \frac{du}{dt} + 2\Omega \cos \varphi \, w - 2\Omega \sin \varphi \, v \tag{2-19a}$$

[2] With $x$, $y$, and $z$ everywhere aligned with the local eastward, northward, and vertical directions, the coordinate system is curvilinear, and additional terms arise in the components of the relative acceleration. These terms are introduced in Section 3-1, to be quickly dismissed because of their relatively small size in most instances.

$$y: \quad \frac{dv}{dt} + 2\Omega \sin \varphi u \tag{2-19b}$$

$$z: \quad \frac{dw}{dt} - 2\Omega \cos \varphi u . \tag{2-19c}$$

For convenience, we define the quantities

$$f = 2\Omega \sin \varphi, \tag{2-20}$$

$$f_* = 2\Omega \cos \varphi . \tag{2-21}$$

The coefficient $f$ is called the *Coriolis parameter*, whereas $f_*$ will be called here the *reciprocal Coriolis parameter*. In the Northern Hemisphere, $f$ is positive; it is zero at the equator and negative in the Southern Hemisphere. In contrast, $f_*$ is positive in both hemispheres, but it vanishes at the poles.

An examination of the relative importance of the various terms (Section 3-4) will reveal that, generally, the $f$-terms are important, whereas the $f_*$-terms can be neglected.

Horizontal, unforced motions are described by

$$\frac{du}{dt} - fv = 0 \tag{2-22a}$$

$$\frac{dv}{dt} + fu = 0 \tag{2-22b}$$

and are still characterized by solution (2-11). The difference resides in the value of $f$, now given by (2-20). Thus, inertial oscillations on the earth have periodicities equal to $2\pi/f = \pi/\Omega \sin \varphi$, ranging from 12 h at the poles to infinity along the equator. Pure inertial oscillations are, however, quite rare because of the usual presence of pressure

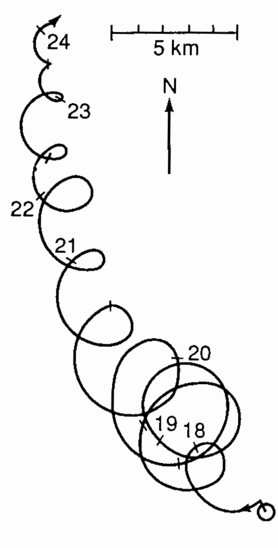


**Figure 2-9** Evidence of inertial oscillations in the Baltic Sea, as reported by Gustafson and Kullenberg (1936). The plot is a progressive-vector diagram constructed by the successive addition of velocity measurements at a fixed location. For weak or uniform velocities, such a curve approximates the trajectory that a particle starting at the point of observation would have followed during the period of observation. Numbers represent days of the months. Note the constant veering to the right, the period of about 14 h, and the value of $2\pi/f$ at that latitude (57.8°N). (From Gustafson and Kullenberg, 1936, as adapted by Gill, 1982.)

gradients and other forces. Nonetheless, inertial oscillations are not uncommonly found to contribute to observations of oceanic currents. An example of such an occurrence, where the inertial oscillations made up almost the entire signal, has been reported by Gustafson and Kullenberg (1936). Current measurements in the Baltic Sea showed periodic oscillations about a mean value. When added to one another to form a so-called progressive vector diagram (Figure 2-9), the currents distinctly showed a mean drift, on which were superimposed quite regular clockwise oscillations. The theory of inertial oscillation predicts clockwise rotation in the Northern Hemisphere at a periodicity of $2\pi/f = \pi/\Omega \sin \varphi$, or 14 h at the latitude of observations, thus confirming the interpretation of the observations as inertial oscillations.

## PROBLEMS

**2-1.** On Jupiter, a day lasts 9.9 Earth hours and the equatorial circumference is 448,600 km. Knowing that the measured gravitational acceleration at the equator is 26.4 m/s$^2$, deduce the true gravitational acceleration and the centrifugal acceleration.

**2-2.** The Japanese Shinkansen train (bullet train) zips from Tokyo to Osaka (both at approximately 35°N) at a speed of 114 mi/h. In the design of the train and tracks, do you think that engineers had to worry about Earth's rotation? (*Hint*: The Coriolis effect induces a transverse force, which could produce a tendency for the train to lean sideways.)

**2-3.** Determine the lateral deflection of a cannonball that is shot in London (51°31′N) and flies for 25 s at an average horizontal speed of 120 m/s. What would be the lateral deflection in Murmansk (68°52′N) and Nairobi (1°18′S)?

**2-4.** On a perfectly smooth and frictionless hockey field at Dartmouth College (43°38′N), how slowly should a puck be driven to perform an inertial circle of diameter equal to the field width (26 m)?

**2-5.** A set of two identical solid particles of mass $M$ attached to each other by a weightless rigid rod of length $L$ are moving on a horizontal rotating plane in the absence of external forces (Figure 2-10). As in geophysical fluid dynamics, ignore the centrifugal force caused by the ambient rotation. Establish the equations governing the motion of the set of particles, derive the most general solution, and discuss its physical implications.

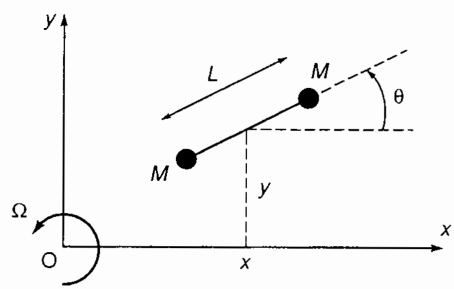


**Figure 2-10** Two linked masses on a rotating plane (Problem 2-5).

**2-6.** Study the trajectory of a free particle of mass $M$ released from a state of rest on a rotating, sloping, rigid plane (Figure 2-11). The angular rotation rate is $\Omega$, and the angle formed by the plane with the horizontal is $\alpha$. Friction and the centrifugal force are negligible. What is the maximum speed acquired by the particle, and what is its maximum downhill displacement?

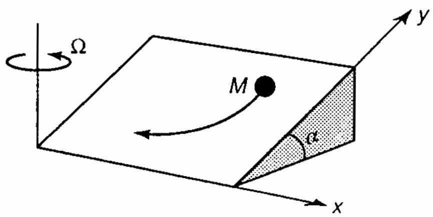


**Figure 2-11** A free particle on a rotating, inclined and frictionless plane (Problem 2-6).

**2-7.** The curve reproduced in Figure 2-12 is a progressive vector diagram constructed from current-meter observations at latitude 43°09′N in the Mediterranean Sea. Under the assumption of a uniform but time-dependent flow field in the vicinity of the mooring, the curve can be interpreted as the trajectory of a water parcel. Using the marks counting the days along the curve, show that this set of observations reveals the presence of inertial oscillations. What is the average orbital velocity in these oscillations?

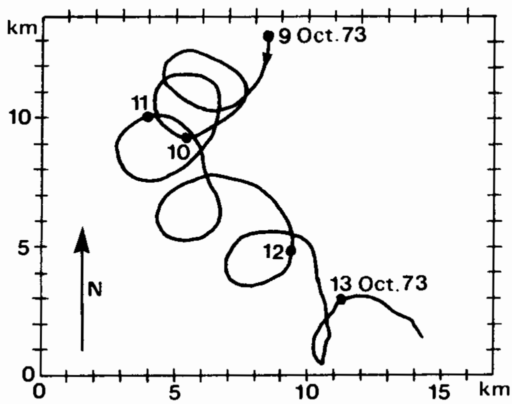


**Figure 2-12** Progressive vector diagram constructed from current-meter observation in the Mediterranean Sea taken in October 1973 (Problem 2-7). (Courtesy of Martin Mork, University of Bergen.)

**2-8.** A stone is dropped from a 300-m-high bridge at 35°N. In which cardinal direction is it deflected under the effect of the earth's rotation? How far from the vertical does the stone land? (Neglect air drag.)

## SUGGESTED LABORATORY DEMONSTRATIONS

### *1. Equipment*

A bare rotating table, a steel ball (approximately 6 mm in diameter), a small wedge with gutter (to help with the release of the ball).

### *Experiment*

Position the wedge on the periphery of the rotating table; facing inward. Place the ball atop the wedge in a stable but precarious manner. Rotate the table; delicately let the ball roll down the wedge; observe the ball's curved path with respect to the rotating table. The visualization is greatly helped if there is a way to record the ball trajectory (by means of chalk dust or a video camera attached to the rotating table).

### *2. Equipment*

A rotating table equipped with a pendulum and a video camera at the top center, looking downward, a monitor screen.

### *Experiment*

Adjust the pendulum's length, $L$, and the rotation rate, $\Omega$, of the table such that $g = L\Omega^2$, where $g$ is the gravitational acceleration (9.81 $m/s^2$). Swing the pendulum and observe its orbit with respect to the rotating table by means of the video camera and remote screen. (*Note*: To simplify the wiring, the video camera can be operated by batteries and the signal transmitted via air waves to the monitor.) Experiment with four kinds of pendulum motions: planar back-and-forth oscillation, azimuthal circling at constant distance from center in the same direction as the rotating table, the same but in a direction opposite to that of rotating table, and elliptical oscillations.

## Gaspard Gustave de Coriolis

### 1792 – 1843

Born in France and trained as an engineer, Gaspard Gustave de Coriolis began a teaching and research career at age 24. Fascinated by problems related to rotating machinery, he was led to derive the equations of motion in a rotating framework of reference. The result of these studies was presented to the Académie des Sciences in the summer of 1831.

The world's largest experimental rotating table, at the Institut de Mécanique in Grenoble, France, is named after him and has been used in countless simulations of geophysical fluid phenomena. (*Photo from the archives of the Académie des Sciences, Paris.*)