

# PART III

## STRATIFICATION EFFECTS

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# 9

## *Stratification*

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**Summary:** After having studied the effects of rotation in homogeneous fluids, we now turn our attention toward the other distinctive feature of geophysical fluid dynamics, namely, stratification. A basic measure of stratification, the Brunt–Väisälä frequency, is introduced, and the accompanying dimensionless ratio, the Froude number, is defined and given a physical interpretation.

### **9-1 INTRODUCTION**

As Chapter 1 stated, problems in geophysical fluid dynamics concern fluid motions with one or both of two attributes, namely, ambient rotation and stratification. In the preceding chapters, attention was devoted exclusively to the effects of rotation, and stratification was avoided by the systematic assumption of a homogeneous fluid. We noted that rotation imparts to the fluid a strong tendency to behave in a columnar fashion—to be vertically rigid.

By contrast, a stratified fluid, consisting of fluid parcels of various densities, will tend under gravity to arrange itself so that the higher densities are found below lower densities. This vertical layering introduces an obvious gradient of properties in the vertical direction, which affects—among other things—the velocity field. Hence, the

vertical rigidity induced by the effects of rotation will be attenuated by the presence of stratification.

Because stratification induces a certain degree of decoupling between the various fluid masses (those of different densities), stratified systems typically contain more degrees of freedom than homogeneous systems, and we anticipate that the presence of stratification permits the existence of additional types of motions. When the stratification is mostly vertical (e.g., layers of various densities stacked on top of one another), gravity waves can be sustained internally (Chapter 10). When the stratification also has a horizontal component, additional waves can be permitted, and, if these grow at the expense of the basic potential energy available in the system, instabilities may arise (Chapter 16).

## 9-2 STATIC STABILITY

Let us first consider fluids in static equilibrium. Such lack of motion requires the absence of lateral forces and, consequently, horizontal homogeneity. Stratification is then purely vertical.

It is intuitively obvious that if the heavier fluid parcels are found below the lighter fluid parcels, the fluid is stable, whereas if heavier parcels lie above lighter ones, the system is apt to overturn, and the fluid is unstable. Let us now verify this intuition. Take a fluid parcel at a height  $z$  above a certain reference level, where the density is  $\rho(z)$ , and displace it vertically to the higher level  $z + h$ , where the ambient density is  $\rho(z + h)$ . If the fluid is incompressible, our displaced parcel retains its former density despite a slight pressure change, and at that new level feels a buoyancy force equal to

$$g[\rho(z + h) - \rho(z)]V,$$

where  $V$  is the volume of the parcel. As it is written, this force is positive if it is directed upward. Newton's law (mass times acceleration equals force) yields

$$\rho(z) V \frac{d^2h}{dt^2} = g[\rho(z + h) - \rho(z)]V. \quad (9-1)$$

Now, geophysical fluids are generally only weakly stratified; the density variations, although sufficient to drive or affect motions, are nonetheless relatively small compared to the average or reference density of the fluid. This remark was the essence of the Boussinesq approximation (Section 3-3). In the present case, this fact allows us to replace  $\rho(z)$  on the left-hand side of (9-1) by the reference  $\rho_0$  and to use a Taylor expansion to approximate the density difference on the right by

$$\rho(z + h) - \rho(z) \simeq \frac{d\rho}{dz} h.$$

After a division by  $V$ , equation (9-1) reduces to

$$\frac{d^2h}{dt^2} - \frac{g}{\rho_0} \frac{d\rho}{dz} h = 0, \quad (9-2)$$

which shows that two cases can arise. The coefficient  $-(g/\rho_0) d\rho/dz$  is either positive or negative. If it is positive ( $d\rho/dz < 0$ , corresponding to a fluid with the greater densities below the lesser densities), we can define the quantity  $N^2$  as

$$N^2 = -\frac{g}{\rho_0} \frac{d\rho}{dz}, \quad (9-3)$$

and the solution to the equation has an oscillatory character, with frequency  $N$ . Physically, this means that, when displaced upward, the parcel is heavier than its surroundings, feels a downward recalling force, falls down, and, in the process, acquires a vertical velocity; upon reaching its original level the particle's inertia causes it to go farther downward and to become surrounded by heavier fluid. The parcel, now buoyant, is recalled upward, and oscillations persist about the equilibrium level. The quantity  $N$ , defined by the square root of (9-3), provides the frequency of the oscillation and can thus be termed the *stratification frequency*. It is, however, more commonly called the Brunt-Väisälä frequency, in recognition of the two scientists who were the first to highlight the importance of this frequency in stratified fluids. (See Brunt's biography at the end of this chapter.)

If the coefficient in equation (9-2) is negative (i.e.,  $d\rho/dz > 0$ , corresponding to a top-heavy fluid configuration), the solution exhibits an exponential growth, a sure sign of instability. The parcel displaced upward is surrounded by heavier fluid, finds itself buoyant, and moves farther and farther away from its initial position. Obviously, small perturbations will ensure not only that the single displaced parcel will depart from its initial position, but that all other fluid parcels will likewise participate in a general overturning of the fluid until it is finally stabilized, with the lighter fluid lying above the heavier fluid. If, however, a permanent destabilization is forced onto the fluid, such as by heating from below or cooling from above, the fluid will remain in constant agitation, a process called *convection*.

In this and the following chapters, we will restrict our attention to stably stratified fluids, for which the stratification frequency,  $N$ , defined from (9-3), exists.

### 9-3 A NOTE ON ATMOSPHERIC STRATIFICATION

In a compressible fluid, such as the air of our planetary atmosphere, density can change in one of two ways: by pressure changes or by internal-energy changes. In the first case, a pressure variation resulting in no internal-energy change (i.e., an adiabatic compression or dilation) is accompanied by both density and temperature variations: All three quantities increase (or decrease) simultaneously, though not in equal proportions. If the fluid is made of fluid parcels all having the same internal-energy content, the lower parcels, feeling the weight of those above them, will be more compressed than those in the upper levels, and the system will appear stratified, with the denser and warmer fluid

underlying the lighter, colder fluid. But such stratification cannot be dynamically relevant, for if parcels are interchanged adiabatically, they adjust their density and temperature according to the local pressure, and the system is left unchanged.

In contrast, internal-energy changes are dynamically important. In the atmosphere, such variations occur because of a heat flux (such as heating in the tropics and cooling at high latitudes, or according to the diurnal cycle) or because of the variations in air composition (such as moisture content). Such variations among fluid parcels do remain despite adiabatic compression/dilation and cause density differences that drive motions. It is thus imperative to distinguish, in a compressible fluid, the density variations that are dynamically relevant from those that are not. Such separation of density variations leads to the concept of potential density.

First, we consider a neutral (adiabatic) atmosphere—that is, one consisting of all air parcels having the same internal energy. Further, let us assume that the air, a mixture of various gases, behaves as a single perfect gas. Under these assumptions, we can write the equation of state and the adiabatic conservation law:

$$p = R\rho T, \quad (9-4)$$

$$\frac{p}{p_0} = \left( \frac{\rho}{\rho_0} \right)^\gamma, \quad (9-5)$$

where  $p$ ,  $\rho$ , and  $T$  are, respectively, the pressure, density, and absolute temperature (in contrast with the preceding chapters, the variables  $p$  and  $\rho$  here denote the full pressure and density);  $R = C_p - C_v$  and  $\gamma = C_p/C_v$  are the constants of a perfect gas. Finally,  $p_0$  and  $\rho_0$  are reference pressure and density characterizing the level of internal energy of the fluid; the corresponding reference temperature  $T_0$  is obtained from (9-4)—that is,  $T_0 = p_0/R\rho_0$ . Expressing both pressure and density in terms of the temperature, we obtain

$$\frac{p}{p_0} = \left( \frac{T}{T_0} \right)^{\gamma/(\gamma-1)} \quad (9-6)$$

$$\frac{\rho}{\rho_0} = \left( \frac{T}{T_0} \right)^{1/(\gamma-1)}. \quad (9-7)$$

Without motion, the atmosphere is in static equilibrium, which requires hydrostatic balance:

$$\frac{dp}{dz} = -\rho g. \quad (9-8)$$

Elimination of  $p$  and  $\rho$  by use of (9-6) and (9-7) yields a single equation for the temperature:

$$\begin{aligned} \frac{dT}{dz} &= -\frac{\gamma-1}{\gamma} \frac{g}{R} \\ &= -\frac{g}{C_p}. \end{aligned} \quad (9-9)$$

In the derivation, it was assumed that  $p_0$ ,  $\rho_0$ , and thus  $T_0$  are not dependent on  $z$ , in agreement with our premise that the atmosphere is composed of parcels with identical internal-energy contents. Equation (9-9) states that the temperature in such atmosphere must decrease with increasing height at the uniform rate  $g/C_p \simeq 10$  K/km. This gradient is called the *adiabatic lapse rate*. Physically, lower parcels are under greater pressure than higher parcels and thus have higher densities and temperatures. This explains why the air is colder on mountain tops than at lower levels.

It almost goes without saying that the departures from this adiabatic lapse rate—and not the total temperature gradients—are to be considered in the study of atmospheric motions. We can demonstrate this clearly by redoing here, with a compressible fluid, the analysis of a vertical displacement performed in the previous section with an incompressible fluid. Consider a vertically stratified gas with pressure, density, and temperature,  $p$ ,  $\rho$ , and  $T$ , varying with height  $z$  but not necessarily according to (9-9); that is, the heat content in the fluid is not uniform. The fluid is in static equilibrium so that equation (9-8) is satisfied. Take a parcel at height  $z$ ; its properties are  $p(z)$ ,  $\rho(z)$ , and  $T(z)$ . Imagine that this fluid parcel is now displaced adiabatically upward over a small distance  $h$ . According to the hydrostatic equation, this results in a pressure change  $\delta p = -\rho gh$ , which causes density and temperature changes given by the adiabatic constraints (9-5) and (9-6):  $\delta\rho = -\rho gh/\gamma RT$  and  $\delta T = -(\gamma - 1)gh/\gamma R$ . Thus, the new density is  $\rho' = \rho + \delta\rho = \rho - \rho gh/\gamma RT$ . But at that new level, the ambient density is given by the stratification:  $\rho(z+h) \simeq \rho(z) + (d\rho/dz)h$ . The displaced parcel experiences an upward force equal to the buoyancy force, which per volume is

$$\begin{aligned} F &= g[\rho_{\text{ambient}} - \rho_{\text{parcel}}] \\ &= g[\rho(z+h) - \rho'] \\ &\simeq g\left(\frac{d\rho}{dz} + \frac{\rho g}{\gamma RT}\right)h. \end{aligned}$$

In terms of the temperature, this force is

$$F \simeq -\frac{\rho g}{T}\left(\frac{dT}{dz} + \frac{g}{C_p}\right)dz.$$

If

$$N^2 = -\frac{g}{\rho}\left(\frac{d\rho}{dz} + \frac{\rho g}{\gamma RT}\right) \quad (9-10a)$$

$$= \frac{g}{T}\left(\frac{dT}{dz} + \frac{g}{C_p}\right) \quad (9-10b)$$

is a positive quantity, this force recalls the particle towards its initial level, and the stratification is stable. As we can clearly see, the relevant quantity is not the total

temperature gradient but the departure from the adiabatic gradient  $g/C_p$ . As in the previous case of a stably stratified incompressible fluid, the quantity  $N$  is the frequency of vertical oscillations. It is called the stratification, or Brunt-Väisälä, frequency.

In order to avoid the systematic subtraction of the adiabatic gradient from the temperature gradient, the concept of potential temperature is introduced. The *potential temperature*, denoted by  $\theta$ , is defined as the temperature that the parcel would have if it were brought adiabatically to a given reference pressure. (In the atmosphere, this reference is usually taken as a nominal ground pressure of 1030 millibars =  $1.03 \times 10^5$  N/m<sup>2</sup>.) From (9-6), we have

$$\frac{p}{p_0} = \left(\frac{T}{\theta}\right)^{\gamma(\gamma-1)}$$

and hence

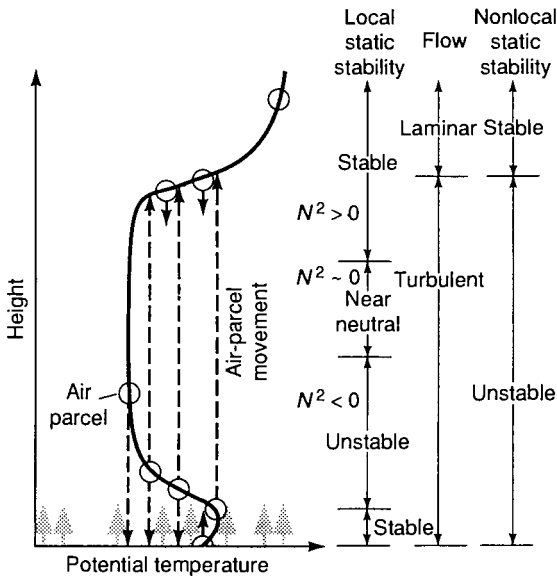
$$\theta = T \left(\frac{p}{p_0}\right)^{-1/\gamma} \tag{9-11}$$

The corresponding density is called the *potential density*, denoted by  $\sigma$ :

$$\sigma = \rho \left(\frac{p}{p_0}\right)^{-1/\gamma} \tag{9-12}$$

The definition of the stratification frequency (9-10a) takes the form

$$N^2 = -\frac{g}{\sigma} \frac{d\sigma}{dz} \tag{9-13}$$



**Figure 9-1** Typical profile of potential temperature in the lower atmosphere above warm ground. Heating from below destabilizes the air, generating convection and turbulence. Note how the convective layer extends not only over the region of negative  $N^2$  but also slightly beyond, where  $N^2$  is positive. Such a situation indicates that a positive value of  $N^2$  may not always be indicative of local stability. (From Stull, 1991.)

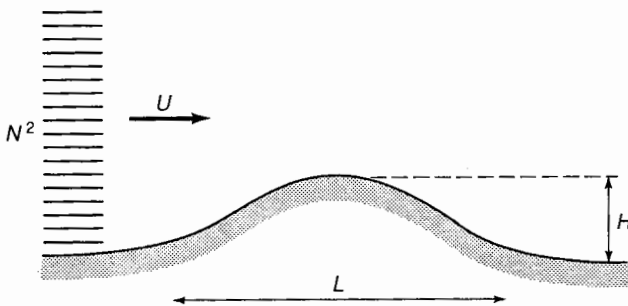
Comparison with the earlier definition, (9-3), immediately shows that the substitution of potential density for density allows us to treat compressible fluids as incompressible.

During daytime and above land, the lower atmosphere is typically heated from below by the warmer ground and is in a state of turbulent convection. The convective layer not only covers the entire region where the time-averaged gradient of potential temperature is negative, but it also penetrates into the region above where it is positive (Figure 9-1). Consequently, the sign of  $N^2$  at a particular level is not unequivocally indicative of stability at that level. For this reason, Stull (1991) advocates the use of a nonlocal criterion to determine static stability. Those considerations apply equally well to the upper ocean under surface cooling.

#### 9-4 THE IMPORTANCE OF STRATIFICATION: THE FROUDE NUMBER

It was established in Section 1-5 that rotational effects are dynamically important when the Rossby number is on the order of unity or less. This number compares the distance traveled horizontally by a fluid parcel during one revolution ( $\sim U/\Omega$ ) with the length scale over which the motions take place ( $L$ ). Rotational effects are important when the former is less than the latter. By analogy, we may ask whether there exists a similar number measuring the importance of the stratification. From the remarks in the preceding sections, we can anticipate that the stratification frequency,  $N$ , and the height scale,  $H$ , of a stratified fluid will play roles similar to those of  $\Omega$  and  $L$  in rotating fluids.

To illustrate how such a dimensionless number can be derived, let us consider a stratified fluid of stratification frequency  $N$  flowing horizontally at a speed  $U$  and encountering an obstacle of width  $L$  and height  $H$  (Figure 9-2). We can think of a wind in the lower atmosphere blowing over a mountain range. The presence of the obstacle



**Figure 9-2** Situation in which a stratified fluid encounters an obstacle, forcing some fluid parcels to move vertically against gravity.

forces some of the fluid to be displaced vertically and, hence, requires some supply of gravitational energy. Stratification will act to restrict or minimize such vertical displacements in some way, forcing the flow to pass around rather than over the obstacle. The greater the restriction, the greater the importance of stratification.

The time passed in the vicinity of the obstacle is approximately the time spent by a fluid parcel to cover the horizontal distance  $L$  at the speed  $U$ , that is,  $T = L/U$ . If the vertical velocity is on the order of  $W$  (to be determined), the corresponding vertical displacements are  $\Delta z = WT = WL/U$ . In the presence of stratification  $\rho(z)$ , these displacements cause density perturbations on the order of

$$\begin{aligned}\Delta\rho &= \left| \frac{d\bar{\rho}}{dz} \right| \Delta z \\ &= \frac{\rho_0 N^2}{g} \frac{WL}{U},\end{aligned}$$

where  $\bar{\rho}(z)$  is the fluid's vertical density profile upstream. In turn, these density variations give rise to pressure disturbances that scale, via the hydrostatic balance, as

$$\begin{aligned}P &= gH\Delta\rho \\ &= \frac{\rho_0 N^2 HLW}{U}.\end{aligned}$$

By virtue of the balance of forces in the horizontal, the pressure-gradient force must be accompanied by a change in the fluid velocity [ $u \partial u / \partial x + v \partial u / \partial y \sim (1/\rho_0) \partial p / \partial x$ ]:

$$U^2 = \frac{P}{\rho_0} = \frac{N^2 HLW}{U}.$$

From this last expression, the ratio of vertical convergence,  $W/H$ , to horizontal divergence,  $U/L$ , is found to be

$$\frac{W/H}{U/L} = \frac{U^2}{N^2 H^2}. \quad (9-14)$$

We immediately note that if  $U$  is less than the product  $NH$ ,  $W/H$  must be less than  $U/L$ , implying that convergence in the vertical cannot fully meet horizontal divergence. Consequently, the fluid is forced to be deflected horizontally so that the term  $\partial u / \partial x$  can be met by  $-\partial v / \partial y$  better than by  $-\partial w / \partial z$ . The stronger the stratification, the smaller is  $U$  compared to  $NH$  and, thus,  $W/H$  compared to  $U/L$ . The stronger the stratification, the weaker the vertical velocity and vertical displacements:

$$\frac{\Delta z}{H} = \frac{WL}{UH} = \frac{U^2}{N^2 H^2}. \quad (9-15)$$

From this argument, we conclude that the ratio

$$Fr = \frac{U}{NH}, \quad (9-16)$$

called the *Froude number*, is the measure of the importance of stratification. The rule is: If  $Fr \lesssim 1$ , stratification effects are important; the smaller  $Fr$ , the more important these effects are.



The analogy with the Rossby number of rotating fluids,

$$Ro = \frac{U}{\Omega L}, \quad (9-17)$$

where  $\Omega$  is the angular rotation rate and  $L$  is the horizontal scale, is immediate. Both Froude and Rossby numbers are ratios of the horizontal velocity scale by a product of frequency and length scale; for stratified fluids, the relevant frequency and length are naturally the stratification frequency and the height scale, whereas in rotating fluids they are, respectively, the rotation rate and the horizontal length scale.

The analogy can be pursued a little further. Just as the Froude number is a measure of the vertical velocity in a stratified fluid [via (9-14)], the Rossby number can be shown to be a measure of the vertical velocity in a rotating fluid. We saw (Section 4-2) that strongly rotating fluids ( $Ro$  nominally zero) allow no convergence of vertical velocity, even in the presence of topography. This results from the absence of horizontal divergence in geostrophic flows (ruling out here, for the sake of the analogy, an eventual beta effect). In reality, the Rossby number cannot be nil, and the flow cannot be purely geostrophic. The nonlinear terms, of relative importance measured by  $Ro$ , yield corrective terms to the geostrophic velocities of the same relative importance. Thus, the horizontal divergence,  $\partial u/\partial x + \partial v/\partial y$ , is not zero but is on the order of  $RoU/L$ . Since the divergence is matched by the vertical divergence,  $-\partial w/\partial z$ , on the order of  $W/H$ , we conclude that

$$\frac{W/H}{U/L} = Ro, \quad (9-18)$$

in rotating fluids. Contrasting (9-14) to (9-18), we note that, with regard to vertical velocities, the square of the Froude number is the analogue of the Rossby number.

In continuation of the analogy, it is tempting to seek the stratified analogue of the Taylor column in rotating fluids. Recall that Taylor columns occur in rapidly rotating fluids ( $Ro = U/\Omega L \ll 1$ ). Let us then ask what happens when a fluid is very stratified ( $Fr = U/NH \ll 1$ ). By virtue of (9-15), the vertical displacements are severely restricted ( $\Delta z \ll H$ ), implying that an obstacle causes the fluid at that level to be deflected almost purely horizontally. (In the absence of rotation, there is no tendency toward vertical rigidity, and parcels at levels above the obstacle can flow straight ahead without much disruption.) If the obstacle occupies the entire width of the domain, such a horizontal detour is not allowed, and the fluid at the level of the obstacle is blocked on both the upstream and downstream sides. This horizontal blocking in stratified fluids is the analogue of the vertical Taylor columns in rotating fluids. Further analogies between homogeneous rotating fluids and stratified nonrotating fluids have been reviewed by Veronis (1967).

### 9-5 COMBINATION OF ROTATION AND STRATIFICATION

In the light of the previous remarks, we are now in position to ask what would happen when, as in actual geophysical fluids, the effects of rotation and stratification are simultaneously present. The preceding analysis remains unchanged, except that we now invoke the geostrophic balance in the horizontal momentum equation to obtain the horizontal velocity scale:

$$\Omega U = \frac{P}{\rho_0 L}.$$

The ratio of the vertical to horizontal convergence then becomes

$$\frac{W/H}{U/L} = \frac{U^2}{N^2 H^2} \frac{\Omega L}{U} = \frac{Fr^2}{Ro}. \quad (9-19)$$

As a result, the influence of rotation ( $Ro \lesssim 1$ ) is to increase the scale for the vertical velocity. However, since a vertical divergence cannot exist without horizontal convergence ( $W/H \lesssim U/L$ ), the following inequality must hold:

$$Fr^2 \lesssim Ro, \quad (9-20)$$

that is,

$$\frac{U}{NH} \lesssim \frac{NH}{\Omega L}. \quad (9-21)$$

This sets an upper bound for the magnitude of the flow field in a fluid under given rotation ( $\Omega$ ) and of given stratification ( $N$ ) in a domain of given dimensions ( $L$ ,  $H$ ). If the velocity is imposed externally (e.g., by an upstream condition), the inequality specifies either the horizontal or the vertical length scales of the possible disturbances. Finally, if the system is such that all quantities are externally imposed and that they do not meet (9-21), then special effects such as Taylor columns or blocking must occur.

Inequality (9-21) brings a new dimensionless number  $NH/\Omega L$ , namely, the ratio of the Rossby and Froude numbers. For historical reasons and also because it is more convenient in some dimensional analyses, the square of this quantity is usually defined:

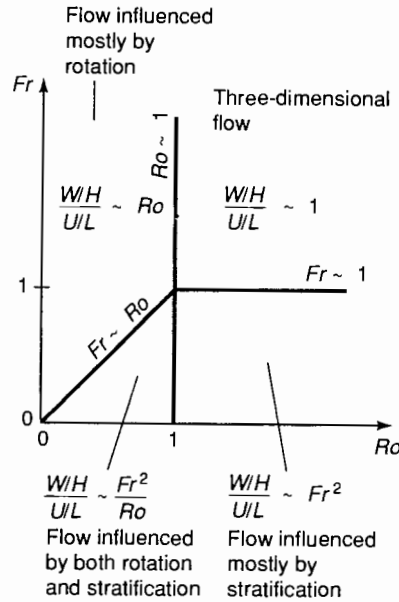
$$Bu = \left( \frac{NH}{\Omega L} \right)^2 = \left( \frac{Ro}{Fr} \right)^2. \quad (9-22)$$

It bears the name of *Burger number*, in honor of Alewyn P. Burger, who contributed to our understanding of geostrophic scales of motions (Burger, 1958). In practice, the Burger number is a useful measure of stratification.

In typical geophysical fluids, the height scale is much less than the horizontal length scale ( $H \ll L$ ), but there is also a disparity between the two frequencies  $\Omega$  and  $N$ . Whereas  $\Omega$ , the rotation rate of the earth, corresponds to a period of 24 h, the stratification frequency generally corresponds to much shorter periods, on the order of

a few to tens of minutes in both the ocean and atmosphere. This implies that generally  $\Omega \ll N$  and opens the possibility of a Burger number on the order of unity.

This is a particular case of great importance. According to our foregoing scaling analysis, the ratio of vertical convergence to horizontal divergence,  $(W/H)/(U/L)$ , is given by  $Fr^2$ ,  $Ro$ , or  $Fr^2/Ro$ , depending on whether vertical motions are controlled by stratification or rotation or both (Figure 9-3). Thus, if  $Fr^2/Ro$  is less than  $Ro$ , stratification restricts vertical motions more than rotation and is the dominant process. The



**Figure 9-3** Recapitulation of the various scalings of the ratio of vertical convergence (divergence) to horizontal divergence (convergence),  $(W/H)/(U/L)$ , as a function of the Rossby and Froude numbers,  $Ro = U/\Omega L$  and  $Fr = U/NH$ .

converse is true if  $Fr^2/Ro$  is greater than  $Ro$ . This relationship implies that stratification and rotation influence the flow field to similar degrees if  $Fr^2/Ro$  and  $Ro$  are on the same order. Such is the case when the Froude number equals the Rossby number and, consequently, the Burger number is unity. The horizontal length scale then assumes a special value:

$$L = \frac{NH}{\Omega} \tag{9-23}$$

For the values of  $\Omega$  and  $N$  just cited and a depth scale  $H$  of 100 m in the ocean and 1 km in the atmosphere, this horizontal length scale is on the order of 50 km and 500 km in the ocean and atmosphere, respectively. At this length scale, stratification and rotation go hand in hand. Later on (Chapter 12), it will be shown that the scale defined above is none other than the so-called internal radius of deformation.

## PROBLEMS

- 9-1. The Gulf Stream waters are characterized by surface temperatures around 22°C. At a depth of 800 m below the Gulf Stream, temperature is only 10°C. Using the value  $2.1 \times 10^{-4} \text{ K}^{-1}$  for the coefficient of thermal expansion, calculate the stratification frequency. What is the horizontal length at which both rotation and stratification play comparable roles? Compare this length scale to the width of the Gulf Stream.
- 9-2. An atmospheric inversion occurs when the temperature increases with altitude, in contrast to the normal situation when the temperature decays with height. This corresponds to a very stable stratification and, hence, to a lack of ventilation (smog, etc.). What is the stratification frequency when the inversion sets in ( $dT/dz = 0$ )? Take  $T = 290 \text{ K}$  and  $C_p = 1005 \text{ m}^2/\text{s}^2 \cdot \text{K}$ .
- 9-3. A meteorological balloon rises through the lower atmosphere, simultaneously measuring temperature and pressure. The reading, transmitted to the ground station where the temperature and pressure are, respectively, 17°C and 1028 millibars, reveals a gradient  $\Delta T/\Delta p$  of 6°C per 100 millibars. Estimate the stratification frequency. If the atmosphere were neutral, what would the reading be?
- 9-4. A wind blowing at a speed of 10 m/s encounters an extinct volcano (of approximately conical shape) 500 m high and 20 km in diameter. The air stratification provides a stratification frequency on the order of  $0.02 \text{ s}^{-1}$ . How do vertical displacements compare to the height of the volcano? What does this imply about the importance of the stratification? Is the Coriolis force important in this case?
- 9-5. Redo Problem 9-3 with the same wind speed and stratification but with a mountain range 1000 m high and 500 km wide.

## SUGGESTED LABORATORY DEMONSTRATION

### *Equipment*

A glass or plexiglass container of arbitrary shape, cranberry juice (or other colored drink with corn syrup or sugar), carbonated water, orange juice (or liquid of intermediate density and different color).

### *Experiment*

Fill the container halfway with cranberry juice. Continue filling, delicately, with carbonated water. To avoid mixing and to create a stratification, pour the water slowly along the sides or through a floating dish with bottom perforations (e.g., an egg carton with holes made at the bottom). Let the liquid sit for a few minutes to allow damping of motions and to permit some diffusion across the fluid interface. Slowly pour (using the same caution and technique) the orange juice and observe the spreading at an intermediate depth, paying particular attention to unsteady motions.



## David Brunt

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**1886 – 1965**

As a bright young British mathematician, David Brunt began a career in astronomy, analyzing the statistics of celestial variables. Then, turning to meteorology during World War I, he became fascinated with weather forecasting and started to apply his statistical methods to atmospheric observations in the search for primary periodicities. By 1925, he had concluded that weather forecasting by extrapolation of cyclical behavior was not possible and turned his attention to the dynamic approach. In 1926 he delivered a lecture at the Royal Meteorological Society on the vertical oscillations of particles in a stratified atmosphere. L. F. Richardson then pointed to a paper published the preceding year by Finnish scientist V. Väisälä, where the same oscillatory frequency was derived. This quantity is now jointly known as the Brunt–Väisälä frequency. Continuing his efforts to explain observed phenomena by physical processes, Brunt contributed significantly to the theories of cyclones and anticyclones and of heat transfer in the atmosphere. His studies culminated in a textbook titled *Physical and Dynamical Meteorology* (1934) and confirmed him as a founder of modern meteorology. (Photo credit: LaFayette, London.)