Homework-04 Due Friday Oct.-14, 2005 (prior to class)

1. The linearized shallow water equations on an f-plane of constant depth H

$$u_t - fv + g\eta_x = 0$$
$$v_t + fu + g\eta_y = 0$$
$$\eta_t + Hu_x + Hv_y = 0$$

allow waves of the form

$$(u,v,\eta) = (u_0,v_0,\eta_0)\exp[i(kx+ly-\omega t)]$$

where (k,l) are wave numbers in (x,y) directions, ω is a wave frequency, $i=\sqrt{-1}$, and subscripts denote differentions. This linear problem can also be written in matrix form as $\underline{\boldsymbol{A}}\bullet(u,v,\eta)=0$ where $\underline{\boldsymbol{A}}$ is a 3×3 matrix of constant coefficients which for nontrivial solutions has a determinant $\det(\underline{\boldsymbol{A}})=0$.

(a) Find the dispersion relation for this class of waves by exploiting $\det(\underline{\boldsymbol{A}})=0$. [5pts]
(b) Find the solutions for velocity (u,v) and show that the velocity vectors describe ellipses with a ratio of minor to major axes is f/ω. [Hint: The problem is greatly simplified by choosing a co-ordinate system oriented in the direction of wave propagation, e.g., assume $\eta=\eta_0\cos(kx-\omega t)$]. [10pt]
(c) Discuss the differences of horizontal current ellipes in the long- and short wave limits, e.g., for $\kappa a \ll 1$ and $\kappa a \gg 1$ where a is Rossby radius of deformation $a=(\sqrt{gH})/f$ and $\kappa=\sqrt{(k^2+l^2)}$. Specifically, comment on the sense of current rotation and ellipticities. [5pts]

2. A wave has the dispersion relation

$$\omega = -\beta_0\, a^2\, k \,/\, [1+a^2(k^2+l^2)]$$

where a=const. is the Rossby radius of deformation, β_0 is the spatial gradient of the Coriolis parameter f, and (k,l) are wavenumbers in the east-west, north-south direction, and ω is the wave frequency:

(a) Find the phase velocities of these waves. [2pts]
(b) Find the group velocties for these waves. [3pts]
(c) Compare and contrast the phase and group velocities in the short and long wave limits. [5pts]

akm-10/11/05