Homework-06 Due Fri Nov.-4, 2005

1. The linear, steady state, shallow water equations on a β-plane of constant depth H

$$-f\rho_0 v = -\partial_x p + \partial_z \tau^{(x)}$$

$$+f\rho_0 u = -\partial_y p + \partial_z \tau^{(y)}$$

$$\partial_x u + \partial_y v + \partial_z w = 0$$

can be written, after vertical integration, as

$$-f\, M^{(y)} = -\partial_x P + \tau^{(x)}$$

$$+f\, M^{(x)} = -\partial_y P + \tau^{(y)}$$

$$\partial_x M^{(x)} + \partial_y M^{(y)} = 0$$

where $M^{(x)} = M_E^{(x)} + M_G^{(x)}$ and $M^{(y)} = M_E^{(y)} + M_G^{(y)}$ are the horizontal components of the total mass flux vectors as the sum of Ekman and geostrophic mass flux vectors indicated by subscripts E and G, respectively.

(a) From the vertically integrated equations, find an expression for the divergence of the Ekman mass flux $\nabla \bullet M_E$ where $M_E = (M_E^{(x)}, M_E^{(y)})$ [5pts]

(b) From the vertically integrated equations, find an expression for the divergence of the geostrophic mass flux$\nabla \bullet M_G$ where $M_G = (M_G^{(x)}, M_G^{(y)})$. [5pts]

(c) Conservation of mass dictates that the divergence of the total mass flux $\nabla \bullet (M_G + M_E) = 0$ which provides you with a strong constraint (and physical insight) of how the geostrophic interior is maintained; interpret this constraint both with respect to to the derivation you just made and relate it to the the vorticity arguments made in class [5pts]

(d) Find expressions for the total mass flux in both x and y directions in terms of a spatially variable wind stress and/or spatial derivatives thereof [5pts]

Note: The governing equations are linear, hence any linear decomposition of the velocity field $u = u_E + u_G$ can be combined to find solutions to the problem. This methodology will not work if the dynamics are non-linear.