The conservation “law”

$$\frac{D_0}{Dt} \{ \zeta_0 + \beta_0 y + \frac{\partial}{\partial z} \left[ \frac{(f_0/N)^2}{\partial \partial z(\psi)} \right] \} = 0$$

is fundamental to the analysis of geophysical fluid motions. Please answer the questions below in carefully written responses:

1. State the fundamental meaning of the equation in physical terms.

2. Summarize the assumptions and restrictions that are required for its validity, emphasizing their physical interpretation. Use non-dimensional parameters where possible, e.g., $Ro<<1$.

3. Give a physical interpretation of the dynamics associated with the operation of the substantial derivative $D_0/Dt$ on each of the three quantities inside the curly braces, e.g., $D_0/Dt(\zeta_0)$.

4. What other dynamical equations are associated with the above equation? State these in their lowest order in $Ro$ and provide a summary of the physical meaning of each.

5. Describe qualitatively how the dynamics represented in the above equation operate for a fluid as it enters the Gulf Stream from the subtropical gyre. (Omit discussion of the shoreward region where the isopycnals approach the surface.) Use a sketch to illustrate your reasoning.

6. Give a simplified form of the above equation applicable to the interior of the subtropical gyres away from frictional boundary layers. Justify your results.

[adapted from an exam given by Dr. Garvine, 1988]