MAST-806 Geophysical Fluid Dynamics Mid-term Exam Oct.-18, 2005 (closed book)

1. [10 pts] The text below is an excerpt of a generally excellent description of Rossby waves posted on the web by Prof. Rhines at the U. Washington. Formally, it contains an tiny error that is better interpreted as an unstated assumption.

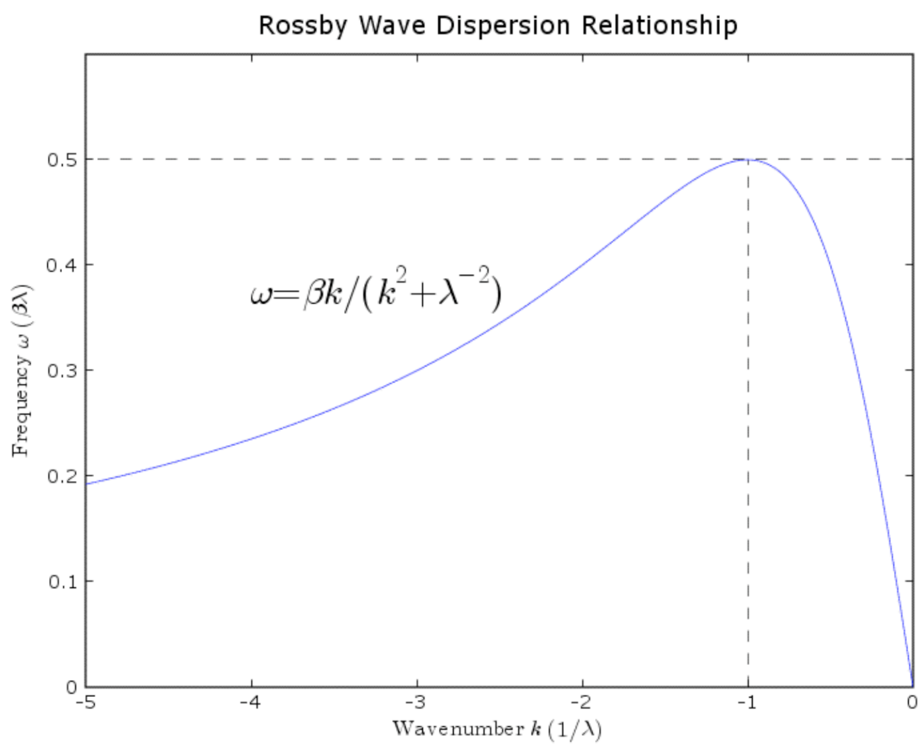
    (a) Find the "error" by deriving the (vertical component of the) vorticity equation that describes this problem in terms of v;
    (b) Rewrite this vorticity equation in terms of sea surface elevation η using both continuity $\eta_t + Hv_y = 0$ and the geostrophic relation for v;
    (c) Find the dispersion relation, compare it against that given in the text below, and thus uncover the implicit assumption made;
    (d) Trace the neglected term from dispersion to vorticity to momentum equations and thus explain physically why this assumption is not a bad one if one is interested in low frequency motions; discuss briefly what type of wave it neglects and what type of wave it describes (properties, restoring).
    (e) Compare magnitude and direction of phase and group velocities for the Rossby wave introduced below.

[Assume that the pressure gradient forces in x- and y-directions are equal to $-g(\eta_x, \eta_y)$ for a barotropic and hydrostatic fluid.]

> Let us now use these ideas to construct a basic Rossby wave for a fluid otherwise at rest. The momentum balance gives us equations in both horizontal directions, x (eastward) and y (northward), for the corresponding velocity components u and v. If we ignore frictional effects and set up a wave with purely north-south motion, u = 0, the momentum equations express an east-west force balance between the pressure gradient and the Coriolis force (the geostrophic balance):
>
> $$-fv = -\frac{1}{\rho} p_x$$
>
> and a north-south force balance between acceleration per unit mass, and pressure gradient:
>
> $$v_t = -\frac{1}{\rho} p_y$$
>
> Eliminating the pressure, p, between these two equations gives a wave equation for v:
>
> $$v_{xt} + \beta v = 0 \qquad (14.1)$$
>
> where β = df/dy, approximated as a constant in the equation. Assuming a wave of the form $v = A\cos(kx - \omega t)$ we substitute in the wave equation to find
>
> $$\sigma = -\beta / k$$
>
> This key relation between the *wavenumber* k (which is 2π divided by the wavelength) and the frequency σ tells us that longer waves have higher frequency and that this frequency scales as $\sigma/f \sim L/a$, the ratio of the length scale of the wave (k^{-1}) to the Earth's radius a. The propagation speed, c, (the phase speed) is westward relative to the fluid, with magnitude β/k^2. In more familiar wave systems, for example *non-dispersive* sound waves, light waves or waves on a vibrating string, the frequency varies directly with wavenumber and the propagation speed c is a constant. As we encountered with surface gravity waves and internal waves, dispersive waves turn a localized disturbance into long trains of sine-waves with gradually varying wavelength (as from a pebble thrown into a pond). By contrast, non-dispersive waves like sound and radio waves preserve the properties of isolated pulses, making possible communication to a great distance.

2. [10 pts] Lets add a constant zonal (east-west) flow U=40 m/s to the problem posed in the text above and derive a dispersion relation that is often used in meteorology to predict weather patterns related to changes in the jetstream and embedded Rossby waves (jetstream meanders):

   (a) Modify the momentum equations for the presence of a steady and spatially uniform zonal flow U [hint: Rossby number Ro~1];
   (b) Eliminate the pressure gradient by deriving the vorticity equation in terms of v (make the same assumptions as the above text does);
   (c) Try wave solutions, as in the text above, for velocity v and thus find a dispersion relation that now involves the constant U;
   (d) Find phase and group velocities; and
   (e) Find the wavelength of a stationary (zero phase velocity), mid-latitude ($\beta \sim 10^{-11}$ m^{-1} s^{-1}) Rossby wave embedded in the uniform flow (U=40 m/s) and comment on the direction of wave phases for waves with shorter and longer wavelengths than those of the stationary Rossby wave.



Rossby radius of deformation:

$$\lambda = \sqrt{gH}/f$$

References:
Text: http://www.ocean.washington.edu/courses/oc513/rhines-AOD-Ch14g-gfd2iii04.pdf
Graph: http://brandt.ocean.washington.edu/OC569/wiki/index.php/Rossby_Waves