MAST602: Introduction to Physical Oceanography (Andreas Münchow)
(Final Exam, Dec.-18, 2008)

1. (10 pts) Write down and discuss the balance of forces and the mass that form the basis of Ekman dynamics.

2. (10 pts) What assumptions must be made for this balance to hold (list at least 5)?

(a)

(b)

(c)

(d)

(e)

3. The solution to the Ekman equations in indefinitely deep water are

$u(z) = V*\exp(z/D)*\cos(z/D)$

$v(z) = V*\exp(z/D)*\sin(z/D)$

Where u and v are the east and north velocity components and z is the vertical co-ordinate counted positive upward, and V is a constant.

3a. (5 pts) Which way does the wind blow?

3b. (5 pts) What is the physical significance of $D=(2A/f)^{1/2}$ where A is the constant vertical eddy viscosity and f the Coriolis parameter? What are its units?

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4. (5pts) What is the net displacement of a particle in a linear wave field over 5 wave periods?

5. (10 pts) Explain the factors and physical causes that determine the number of intense western boundary currents that may exist in a closed ocean basin. Sketch the wind field for an ocean basin in the northern hemisphere with 5 such currents.

6. (10 pts) How can you reconcile the fact that the gravitational attractive force of the moon that causes a bulge towards the moon also causes a bulge away from the moon on the side of the earth facing away from the moon?

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7. (15 pts) Please edit the following text to make it physically sound and accurate:

Kelvin waves in the northern hemisphere propagate along the coast as shallow water waves with a phase speed $c=g/\kappa$ where g is the constant of gravity and $\kappa=2\pi/\lambda$ is the wave number related to the wavelength λ. Kelvin waves are non-dispersive, i.e., their phase velocity equals their group velocity, however, shorter waves travel more slowly that longer waves. They can travel only in the direction with the coast to its left looking in the direction of wave phase propagation. The wave form of a Kelvin wave is peculiar since at any instant in time sea level oscillations occur both along and across the shelf. This is the result of a geostrophic force balance in the along-shore direction while acceleration balances the pressure gradient in the across-shore direction. Friction is generally negligible for these waves except adjacent to the coast. Here, the Kelvin waves result from a frictional or Ekman balance.

Surface Kelvin waves as described above are generated by ocean tides and local wind effects in coastal areas. For example a storm off Nova Scotia can send a Kelvin wave that follows the shores of eastern Canada and the the eastern US seaboard eventually reaching Cape Hatteras. Traveling in approximiate 40 m of water over a distance of 2200 km, it accomplishes its journey in about 31 hours.

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8. (30 pts) In the 1970ies and 1980ies coastal oceanographers discovered observational evidence on the existence of a previously hypothesized class of waves called topographic Rossby or continental shelf waves. Similar to Rossby waves, these peculiar waves originate from a linear balance between local acceleration, Coriolis acceleration, and pressure gradients as well as continuity of mass, that is,

East-west momentum balance:	$\partial u/\partial t - fv = -g\ \partial\eta/\partial x$
North-south momentum balance:	$\partial v/\partial t + fu = -g\ \partial\eta/\partial y$
Continuity:	$\partial\eta/\partial t + H\ (\partial u/\partial x+\partial v/\partial y) = 0$

However, unlike Rossby waves the Coriolis parameter f is now a constant, e.g., $1\times10^{-4}\ s^{-1}$, but the bottom depth varies. Without loss of generality, let us consider the bottom to vary in the north-south direction as $H(y)=H_0-\alpha y$ with deep water at y=0 having H_0=500-m sloping upward towards the north with a slope $\alpha=10^{-2}$, a setup that perhaps mimics the shelf break in the Gulf of Mexico between Texas and Alabama.

A number of peculiar properties can be inferred from its dispersion relation:

$$\boldsymbol{\sigma = -\ \alpha\ g/f\ \ [\kappa/\ (1+\kappa^2\ R^2)]}$$

where σ is the wave frequency, κ is the wave number in the east-west direction, $R=(g\ H_0)^{1/2}/f$ is the constant Rossby radius of deformation, and $g=9.81\ m/s^2$ is the constant of gravity.

Your task here is to **describe as many physical characteristics of this so-called topographic shelf wave as possible** based on this dispersion relation.

Please be careful to introduce a co-ordinate system, sketch the geometry of the problem, and recall that the sign of (x,y) components of phase and group velocities indicate the direction of wave propagation and energy flux in (x,y) directions, respectively.

Note that the wavelength $\lambda=2\pi/\kappa$ of these so-called topographic Rossby waves compares against a horizontal length scale R. Therefore, you can distinguish between waves that are long and short relative to R.

[Key words: wavelength, phase velocity, group velocity, direction of propagation, dispersion, short waves, long waves, Rossby radius, time and space scales]