Rossby Wave Exercise Oct. 21, 2008 1

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$$\sigma = \frac{-\beta\kappa}{(\kappa^2 + R^{-2})} = -\beta R^2 \frac{\kappa}{1+(\kappa R)^2} = \begin{cases} -\beta R^2 \kappa & \kappa R \ll 1 \\ -\beta/\kappa & \kappa R \gg 1 \end{cases}$$

$$c = \sigma/\kappa = -\beta R^2 \frac{1}{1+(\kappa R)^2} = \begin{cases} -\beta R^2 & \kappa R \ll 1 \\ -\beta/\kappa^2 & \kappa R \gg 1 \end{cases}$$

$$c_g = \frac{\partial\sigma}{\partial k} = \begin{cases} -\beta R^2 & \kappa R \ll 1 \\ +\beta/\kappa^2 & \kappa R \gg 1 \end{cases}$$

$$\frac{\partial}{\partial x}\left(\frac{1}{x}\right) = \frac{0 \cdot x - 1 \cdot 1}{x^2} = -\frac{1}{x^2}$$

$\kappa R \ll 1$ means $R \ll \frac{\lambda}{2\pi}$ long waves

$\kappa R \gg 1$ means $R \gg \lambda/2\pi$ short waves

Waves long relative to the deformation radius

1. non-dispersive $\longrightarrow c_p = c_g = -\beta R^2$ same speed for all waves

Waves short relative to the deformation radius

2. dispersive $\longrightarrow c_p = -c_g = -\beta/\kappa^2 = -\beta \frac{\lambda^2}{4\pi}$

longer waves propagate faster

The negative sign in the phase speed means that these waves <u>always</u> propagate their phase to the west, they cannot propagate eastward.

The group velocity for the dispersive shorter waves, however, is in the opposite, that is, always eastward direction.

Short here means that their wavelengths are short relative to the Rossby radius R.

$$R = \sqrt{g'H}/f \approx \frac{\sqrt{10^{-2}\cdot 10^{3}}}{10^{-4}} \approx 3\cdot 10^{4}\ \mathrm{m} \approx 30\ \mathrm{km}$$

Pick a long non-dispersive wave that, say, has a wavelength

$\lambda \gg R \qquad \lambda \sim 300$ km

This wave has a frequency of $\sigma \sim -\beta R^2 k$

$$\cancel{T \sim -\beta R^2 \lambda = 2\cdot 10^{-11}\cdot \overset{3\cdot 10^4\cdot 3\cdot 10^4}{\cancel{}}\cdot 3\cdot 10^5 = 54\cdot 10^{-11}\cdot 10^{8}\cdot 10^{5} = 54\cdot 10^{2}\ \mathrm{s} = 5400\ \mathrm{s} \approx 2\ \text{hours}\ (?)}$$

$$\frac{\cancel{2\pi}}{T} \sim -\beta R^2 \frac{\cancel{2\pi}}{\lambda}$$

$\Rightarrow$

$$T \sim \frac{\lambda}{\beta R^2} = \frac{3\cdot 10^5}{2\cdot 10^{-11}\cdot 9\cdot 10^{8}} = \frac{3}{18}\cdot 10^{8} = \frac{1}{6}\cdot 1200 \quad \cancel{\approx 3\ \text{years}}$$

≈ 200 days

Long Wave $\quad \lambda \sim 300$ km $\qquad T \sim 200$ day

$$c = -\beta R^2 \approx 2 \cdot 10^{-11} \cdot 9 \cdot 10^8 = 18 \cdot 10^{-3} \text{ m/s}$$
$$\approx 1.8 \cdot 10^{-2} \text{ m/s}$$
$$= 1.8 \text{ cm/s}$$

Short waves $\quad \lambda \ll R \qquad \lambda = 3$ km

$$\frac{R\pi}{T} \approx \frac{-\beta}{2\pi} \lambda$$

$$\text{s} \quad T \approx \frac{4\pi^2}{\beta \lambda} = \frac{4\pi^2}{2 \cdot 10^{-11} \cdot 3 \cdot 10^4} \approx \frac{40}{\cancel{6}} \cdot 10^{+7}$$

$$\approx \cancel{6} \cdot 10^7 \text{ s} = \cancel{6} \cdot 115 \text{ days} \approx 2 \text{ years}$$

$$C_p \approx \frac{-\beta}{k^2} = \frac{-\beta}{2\pi} \cdot \lambda^2 = \frac{2 \cdot 10^{-11}}{\cancel{6}} \cdot \cancel{9} \cdot 10^6 = 3 \cdot 10^{-5} \frac{\text{m}}{\text{s}}$$

$$= 3 \cdot 10^{-3} \text{ cm/s}$$

almost stationary, does not move at all, in-place

All ~~Long~~ - Wave propagate phase westward, energy of long non-dispersive waves propagates westward also, but short waves propagate their energy eastward (very slowly, though)

$$\sigma = \frac{-\beta\kappa}{\kappa^2 + R^{-2}} = \sigma(\kappa)$$

$$\frac{d\sigma}{d\kappa} = \frac{d(-\beta\kappa)}{d\kappa} \cdot (\kappa^2+R^{-2})^{-1} + (-\beta\kappa) \cdot \frac{d}{d\kappa}\left(\frac{1}{\kappa^2+R^{-2}}\right)$$

$$= -\beta(\kappa^2+R^{-2})^{-1} + (-\beta\kappa) \cdot \left(-\frac{2\kappa}{(\kappa^2+R^{-2})^2}\right)$$

$$\cancel{d\sigma}\ \frac{d\sigma}{d\kappa} = -\beta(\kappa^2+R^{-2})^{-1} + \frac{2\beta\kappa^2}{(\kappa^2+R^{-2})^2}$$

$$\frac{d\sigma}{d\kappa} \overset{!}{=} 0 \quad : \quad 0 = -\beta + \frac{2\beta\kappa^2}{\kappa^2+R^{-2}}$$

$$\cancel{\kappa^2+R^{-2}}\,0 = \cancel{-\beta\kappa^2} - \beta R^{-2} \cancel{+2\beta\kappa^2}$$

$$\cancel{\kappa^2}\,0 = \beta\kappa^2 - \beta R^{-2}$$

$$\kappa^2 = +R^{-2}$$

$$\kappa = R^{-1}$$

$$\frac{2\pi}{\lambda} = R^{-1}$$

$$\lambda = 2\pi R$$

wavelength of zero group velocity

At this wavelength the wave has its largest possible frequency

$$\sigma(\kappa = R^{-1}) = \frac{-\beta R^{-1}}{2R^{-2}} = -\frac{\beta R}{2} = \frac{\cancel{2}\cdot 10^{-11}}{\cancel{2}\ \cancel{m}\, s} \cdot 3.5\cdot 10^4\, \cancel{m} = 3.5\cdot 10^{-7}$$

$$T = \frac{2\pi}{\sigma} = \frac{2\pi}{3.5}\, 10^7\, s \approx \pi \cdot 115\ \text{days} \approx 1\ \text{year}\ ??$$